Implementation and validation of a new method to model voluntary departures from emergency departments

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Abstract

In literature, several organizational solutions are proposed to determine the probability of patients' voluntary discharge from the emergency department. Here, the issue of self-discharge is analyzed by modeling through the Markov theory, an innovative approach recently applied to the healthcare field. The aim of the work is to propose a new method to calculate the rate of voluntary discharge by defining a generic model to describe the process of first aid using the "behavioral" Markov chain model, a new approach that takes into account the satisfaction of the patient. The proposed model is then applied on MatLab and validated to a real case study at the hospital "A. Cardarelli" of Naples. It was found that most of the risk of self-discharge is during the wait time before the patient is seen and for the final report; usually, once the analysis is requested, the patient, although not very satisfied, is willing to wait longer for the results. The model allows the description of the first aid process from the perspective of the patient. The presented model is generic and adaptable to each hospital facility by changing only the transition probabilities between states.

KEYWORDS

Markov Chain; Emergency Department; Simulation; Voluntary Departure

INTRODUCTION

In recent decades, the subject of health management has aroused interest from political and social points of view [1], since the conspicuous consumption and waste of financial and human resources dedicated to the public health results in low quality and poor access to the healthcare services.

As shown by the studies of Nulty and Ferlie [2], healthcare management takes shape in a highly political and complex organizational context, characterized by the coexistence of different professional groups and numerous regulatory systems. These features make it difficult to apply to the healthcare system those management techniques that have been successfully employed in other areas [3-4].

In particular, hospital emergency departments (ED) consist of complex organizational structures that can be analyzed, managed and improved by pursuing the objectives of effectiveness, efficiency, reduction of waste and resources. Good management of the hospital ED is crucial because it is the interface between resources and health [5-6].

Emergency Department (ED) are facing to lack of resources, long wait times, overuse of emergency services. A lot of patients decide during their permanence into the ED to leave without being seen [7]. This problem underlines the inefficiencies bottlenecks constraints and risks. The concept of urgency is the most important concept in the emergency medicine [8]. The degree of urgency is influenced by different elements such as societal and organizational factors too. When the degree of medical urgency is assessed by a care team involving different categories of staff (physician, nurse, and nursing assistant or secretary) this is called team triage. This flow process can shorten the time before initial contact with a physician and shorten the length of stay in the ED. Team triage also leads to fewer patients spontaneously leaving the ED before they have been medically evaluated. A lot of different methods are discussed in order to limited this problem, such as streaming [9], the process in which patients with similar problems were allocated to a particular work stream until to the use of Point-of-care testing (POCT) which allows to move laboratory standard testing into the ED increasing the speed of diagnosis [10]. Furthermore, different methods are used in order to forecast the flow and to care pathway for patients in a healthcare ED. In 2009, Masso et al. [11] identified, thanks to the use of same questionnaire, a large number of reasons for potential primary care cases presenting to the ED than the patients themselves report. In 2011, Weng et al. [12] find out an optimal allocation of resources in ED using the simulation to smoothen the flow of ED. The simulation model was according with National Emergency Department Overcrowding Scale (NEDOCS) and OptQuest in Simul8. Abo-Hamad et al. [13] proposed an integrated framework to analyze patient flow through an ED. In this work [13], the authors used a simulation model based on decision support framework. Di Leva et al. [14] used Business Process – Modeling (BP-M) in order to be able to run different scenarios, to identify and try to find a solution to the bottlenecks. Differently from what previously presented, same authors used Markov Chain in order to analyze the patient flow in ED [15]. In 2013, Zhu et al. [16] used this method in order to forecast patient flow, the authors focus on expounding the principle of this model and comparing real-word results. In this contest, the occupancy level can be modelled as Markov Chains (MC) on a week to week bases and can be presented as Markov Chain Decision Processes. This method can be applied and adapted to policies of different hospitals and it could be extended in order to include more states and actions to capture and represent more accurately the real difficulties of everyday.

The ED plays a vital role in providing first aid to patients and often faces several problems such as maintenance costs, patient safety and processing [17-22], overcrowding and delays [23-26]. First aid activities are treated as components of a process to describe, map, analyze, and improve the perspective of quality, understood as patient satisfaction. However, management of an ED is complicated because of the peculiar activities that constitute the aid process, whose final goal is the discharge of a satisfied patient. The most difficult ED procedures to deal with are identification, acceptance, and monitoring of patient handling [27-28].

The patients' discharge from an ED without receiving examinations, i.e. patients going to the ED for a medical treatment but leaving without receiving evaluation by a doctor, represents a considerable indication of low quality and malfunction of the department [29-31]. The voluntary dropout rate can be considered one of the most important indicators of crowding in the ED and, in general, of the perception of quality. Patients claiming to have left the department because of dissatisfaction blame the long waiting time. In summary, when patients wait longer than they expect to receive medical examination in an ED, the patient satisfaction decreases and the leaving without being seen (LWBS) frequency rate increases [32-34].

Within this framework, we introduce the importance of deepening the modern techniques to manage processes, enhancing the critical issues and problems that must be solved to achieve continuous improvement and waste reduction. In particular, this work is intended to establish the probability of voluntary departure from the emergency department in the hospital starting from an assessment of the level of the patients' satisfaction. Indeed, dissatisfaction due to the long wait and the lack of medical attention can lead to LWBS [35]. Thus, by assessing the probability that the patient will reach a certain level of dissatisfaction, the probability of voluntary departure from the ED can be evaluated. To do so, we modelled the first aid process by using the MC theory, taking into account the satisfaction of the patient.

METHODS

Markov chain models

MC theory is a theoretical tool that describes the process and calculation of the self-discharge rate. With Markovian theories, events are represented as transitions among the different states, and they can be shown in a matrix representation. In this context, the aid process can be taken as a model through Markovian chains in which states represent the various process steps, and transitions among them are defined according to the odds of passing to a following stage.

Among various techniques and approaches, Markov models have been shown their usefulness and applicability in several contexts. For production planning and control, the research of risk models in literature often involves the use of Markovian theories [36-37]. Furthermore, as previously said, the Markov models are useful in several situations and in healthcare too. Representing clinical situations with traditional decision trees is difficult and might require the simplification of unrealistic assumptions. Usually Markov models suppose that the patient, during his or her course through hospital facilities, is in one of the states of the process, called states of Markov [38-40].

During the last decade, Markov chain Monte Carlo (MCMC) techniques have revolutionized Bayesian statistics providing to the statisticians the ability to perform inference in realistically complex stochastic models. In theory, MCMC techniques allow Bayesian statisticians to sample from posterior distributions in nearly all statistical models of applied interest [41-42].

In Bayesian statistic, the operation of integration has a very important role indeed, as an example, given a sample $\mathbf{y}$ from a distribution with likelihood L ($\mathbf{y}$ |x) and a prior density for $\mathbf{x}$ \euro R$^{\mathrm{p}}$ given by p($\mathbf{x}$), Bayes's theorem relates the posterior π($\mathbf{x}$ |$\mathbf{y}$) to the prior *via* the formula

π ($\mathbf{x}$ |$\mathbf{y}$) αL ($\mathbf{y}$ |$\mathbf{x}$) p($\mathbf{x}$) (1)

where the constant of proportionality is given by

[?] L ($\mathbf{y}$ |x) p(x) dx. (2)

Given the posterior, and in the case where$\mathbf{x}$ =($\mathbf{x}_{1}$, $\mathbf{x}_{2}$) is multivariate, for example, we may be interested in the marginal posterior distributions, such as

π ($\mathbf{x}_{1}$|$\mathbf{y}$) = [?] π ($\mathbf{x}_{1}$,$\mathbf{x}_{2}$|$\mathbf{y}$) d$\mathbf{x}_{2}$ (3)

Alternatively, we might be interested in summary inferences in the form of posterior expectations, e.g.

E (ϑ(x) /$\mathbf{y}$)= [?] ϑ(x) π (x|$\mathbf{y}$) dx. (4)

Thus, for calculating the normalizing constant in expression (2), the marginal distribution in equation (3) or the expectation in equation (4) is important to be able to integrate often complex and high dimensional functions.

An explicit evaluation of these integrals is often not possible, and the solution could be the use of the numerical integration or the analytic approximation techniques. However, the Markov chain Monte Carlo method may be used as an alternative sampling from the posterior directly, and obtaining sample estimates of the quantities of interest, thereby performing the integration implicitly.

The common undergraduate approach to Markov chain theory, is to start with some transition distribution modelling some process of interest, to determine conditions under which there is an invariant or stationary distribution and then to identify the form of that limiting distribution. MCMC methods involve the solution of the inverse of this problem whereby the stationary distribution is known, and it is the transition distribution that needs to be identified, though in practice there may be infinitely many distributions to choose from.

The main theorem underpinning the MCMC method is that any chain which is irreducible and aperiodic will have a unique stationary distribution, and that the t-step transition Kernel will 'converge' to that stationary distribution as t-[?]. Thus, to generate a chain with stationary distribution π, we need only to find transition Kernels K that satisfy these conditions and for which π K =π, i.e. K is such that, given an observation $\mathbf{x}$ π($\mathbf{x}$), if $\mathbf{y}$ K ($\mathbf{x}$, $\mathbf{y}$), then $\mathbf{y}$ π($\mathbf{y}$), also.

In practice, it is often most sensible to combine a number of different transition Kernels to construct a Markov chain which performs well [41].

Numerous are the biomedical application of the MC. In a recent publication *Roy et al.* faced the problem of predicting risk of adverse events (AEs) following surgical procedure [43]. This topic is of significant interest, as that may guide in better resource utilization and an improved quality of care. They propose a study to improve the current techniques for assessing and predicting the risk of adverse events associated with multiple chronic conditions by designing machine learning models that account for and incorporate the temporal sequence and timing of conditions. Their technical addition relay on devising novel sequence-based feature discovery techniques to improve existing supervised classification algorithms, as well as formalizing the classification task as a MC model that captures the temporal sequence of prior chronic conditions/events. There are many examples of machine learning studies to help clinicians in the phases of both diagnosis and prognosis: in neurology [44-45], in cardiology [46-48], in radiology [49-50] and in the oncologic field [51].

Biophysical multi-compartment models, used in diffusion MRI analysis, require parameter estimation, typically performed using either Maximum Likelihood Estimation (MLE) or using MCMC sampling. Whereas MLE provides only a point estimate of the fitted model parameters, MCMC recovers the entire posterior distribution of the model parameters given the data, providing additional information such as parameter uncertainty and correlations. Harms *et al.* [52] investigated the performance of MCMC algorithm variations over multiple popular diffusion microstructure models to see whether a single well performing variation could be applied efficiently and robustly to many models.

In their manuscript, Yuan [53] introduces a novel method to predict protein subcellular locations from sequences. Using sequence data, this method achieved a prediction accuracy higher than previous methods based on the amino acid composition. The study used Markov chains to predict protein subcellular locations. In particular, they applied a first-order Markov chain and extended the residue pair probability to higher-order models. This method achieved a prediction accuracy that was 8% higher than the neural networks method, based on the amino acid composition.

Markov models have been also used to model the patients flow in ED [15, 16, 54]. These models are also suitable to represent the patient admission decision at an emergency department [55-57] or the admission control problem of patients arriving at an emergency department in the aftermath of a mass casualty incident or disasters and crisis management plan [58]. In addition, the patient transitions between emergency department, intensive or critical care unit, and hospital ward have been used with a Markov chain model [59].

Markov processes and their properties

Given a set of states S = [s_1, $s_2 \ldots s_n$], the process begins in one of these states and subsequently moves from one state to another. Every movement is called a step. If the chain is currently in the s_i state and then moves toward s_j to the next step with a probability defined as asp_{ij}, this probability does not depend on the State in which the chain was before the current one.

Probability p_{ij} is called transition probabilities. The process may remain in the state in which it is located with a probability p_{ii}. An initial distribution of probability, defined on S, specifies the state of departure. Normally, the State of departure is a particular State.

Three elements are to be specified in characterizing a Markov Chain (MC):

- The state space S: S is a finite or countable set of States, which are the values that the aleatory variables X_i can take. We label the states as follows: S = [1.2, ... N] for N finite, or S = [1.2, ...] to the case when N = [?].
- The initial distribution π_0: This is the probability distribution of the MC at zero time. For each of the states i S, it will be indicated by π_0 (i) the probability P [X_0 = i], which, with the MC, you initialize the State. Formally, π_0 is a function that has values in the range [0.1] such that π_0 $\pi_0 \geq 0 \forall\ i\ \in S$ and $\sum_{i \in S} \pi_0\ (i) = 1$.

Equivalently, instead of considering π_0 as a function from S going in [0.1], you can think of it as a vector whose i th component is equal to the probability π_0(i) = P (X_0= i). The rule of transition probabilities is specified by providing a matrix P = (P_{ij}). If S contains a number N, then P will be a matrix N x N. P_{ij} can be interpreted as the conditional probability: the probability that the chain moves instantly to $n + 1$ j status as it is in the instantly n ; or P_{ij} = P (X_{n+1} = j | X_n= i).

Formally, the transition matrix is a square matrix N x N whose elements are non-negative and whose rows have a unitary sum. Finally, you may be wondering why you should organize these probabilities within an array.

In the case of the aid process, states represent the various process steps, and transitions among them are defined according to the odds of passing to a following stage.

The proposed Markovian approach

The proposed approach follows these three steps:

1. Firstly, we constructed a description of the process, i.e. the path of the patient from the initial identification to the discharge, starting from the MC flowchart as reported elsewhere. The system provides a vector containing the probability of a patient's self-discharge.
2. Then, we obtained a more complex representation of the process, called the "behavioral chain", which takes into consideration the level of patient's satisfaction. The approach is the same used for MC elsewhere [60-61]: states of satisfaction are achieved with the transition probabilities that depend on waiting times. To define the time limits within which the patient feels satisfied, not very satisfied, and dissatisfied, we addressed the patients by using a questionnaire while they waited in the ED.
3. Finally, the Markov model is validated based on data supplied by the "Centro di Elaborazione Dati" (CED) of the hospital "A. Cardarelli" of Naples and is simulated on MatLab [62].

The transition probabilities of the MC model have been calculated on the basis of the data provided by the company's arrival rates CED and waiting time. The data are reported in Table 1.

Table 1. Rates of arrival

Priority Code	Medical resource	Orthopedic resource	Surgical resource	Total
White Code	181	58	18	257
Green Code	21,703	7909	7873	37,485
Yellow Code	21,919	262	1832	24,013
Red Code	717	7	114	838
TOTAL	44,520	8236	9837	62,593

Transition probabilities related to the number of patients who underwent laboratory tests and/or radiographic findings were obtained by considering data and percentages reported in Table 2.

Table 2. Rates of patients with analysis and examinations in three different cases: surgical, medical and orthopedic

Priority Code	**Laboratory analysis**	**%**	**Diagnostic imaging**	**%**	**No analysis**
Surgical case	*Surgical case*	*Surgical case*	*Surgical case*	*Surgical case*	*Surgical case*
White Code	0	0	4	0.22	14
Green Code	180	0.02	4394	0.56	3299
Yellow Code	267	0.15	1292	0.70	273
Red Code	43	0.38	71	0.62	0
TOTAL	**490**	**0.05**	**5761**	**0.59**	**3586**
Medical case	*Medical case*	*Medical case*	*Medical case*	*Medical case*	*Medical case*
White Code	0	0	4	0.22	14
Green Code	180	0.02	4394	0.56	3299
Yellow Code	267	0.15	1292	0.70	273
Red Code	43	0.38	71	0.62	0
TOTAL	**490**	**0.05**	**5761**	**0.59**	**3586**
Orthopedic case	*Orthopedic case*	*Orthopedic case*	*Orthopedic case*	*Orthopedic case*	*Orthopedic case*
White Code	0	0	9	0.16	49
Green Code	10	0.01	3100	0.39	4799
Yellow Code	7	0.03	133	0.51	122
Red Code	1	0.14	1	0.14	5
TOTAL	**18**	**0.01**	**3243**	**0.39**	**4975**

Both the validation and the calculation of the distribution limit of the chains were carried out by means of MatLab Software. An analysis of alternative scenarios to assess the potential impact of quality improvements in the process is also provided.

RESULTS

First aid process modeling with flowchart

Figure 1 illustrates the aid process flow diagram, which highlights the events of a patient's voluntary self-discharge from the ED, from the patient's identification by triage nurses to the arrival at the ED.

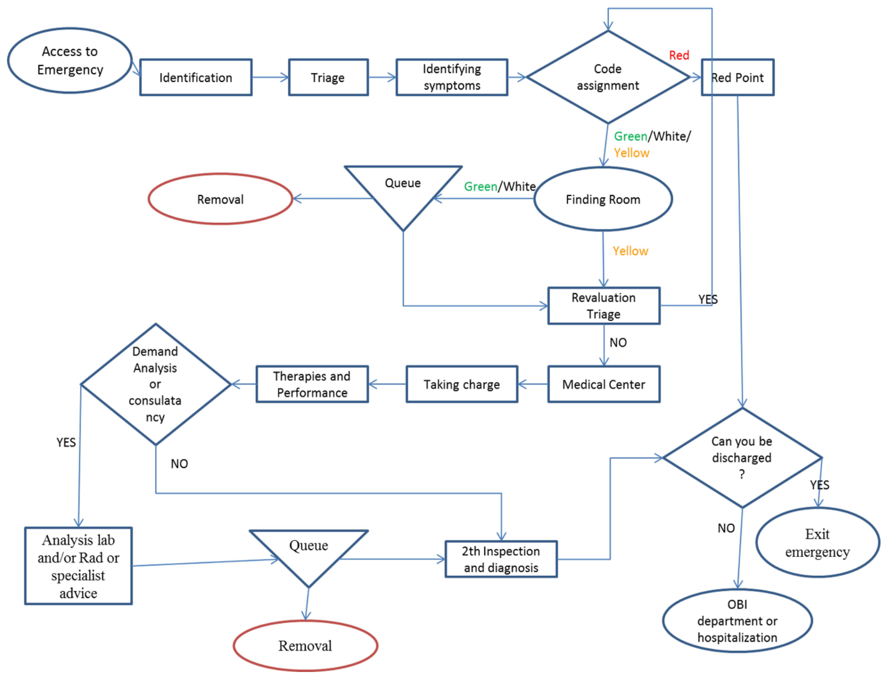


Figure 1. First aid flow chart.

Nurses, based on the analysis of vital signs (consciousness, breathing, and blood circulation), assign to each patient a priority code and a medical resource (medical, surgical, orthopedic), depending on the urgency and type of disease. If the patient presents deficiency in one of the three vital signs, a red code is assigned, requiring immediate action; the patient is immediately led to a dedicated area (called Red Point) where he or she has the right of first refusal, or service outage for patients of any other code whenever resources are all busy. If the patient shows signs that medical help may be delayed, he or she will be assigned a priority code of yellow, green, or white and will be led back to the waiting room. If necessary, patients receive first aid (such as a bandage) or undergo some initial checks before entering the queue and waiting in a room monitored by triage nurses, so that any increase in severity can be detected immediately. As soon as resources are available and according to priority, patients receive a first medical consultation, following a first in first out (FIFO) rule for that color code. During this first inspection, the staff establishes the patient's condition and assigns a treatment.

Then the patient has to decide between the request for further analysis (laboratory tests such as urinalysis or hematologic–chemical tests such as X-rays and ultrasound) and discharge or hospitalization. This step is followed by a second inspection, after waiting for the analyses' results, and finally decides whether to be discharged or admitted to the hospital. Due to overcrowding, congestion, or unavailability of resources, waiting times can be long, thereby generating patient dissatisfaction and a voluntary and untraceable discharge from the ED. Normally, the main cause for waiting in the emergency department is obtaining test results, particularly from the laboratory. Waiting times also vary, depending on priority and type of resource consumption; the lowest priority codes tend to wait longer because they are judged less urgent.

12 example cases can occur: white orthopedic, surgical, or medical code; green orthopedic, surgical, or medical code; yellow orthopedic, surgical, or medical code; red surgical, medical, or orthopedic code. The highest priority code (red) should not wait long because, as has already been explained, the patient has the right of first refusal over other patients.

Emergency Department processes

The chain in Figure 2 shows where patients can be found in the emergency department process.

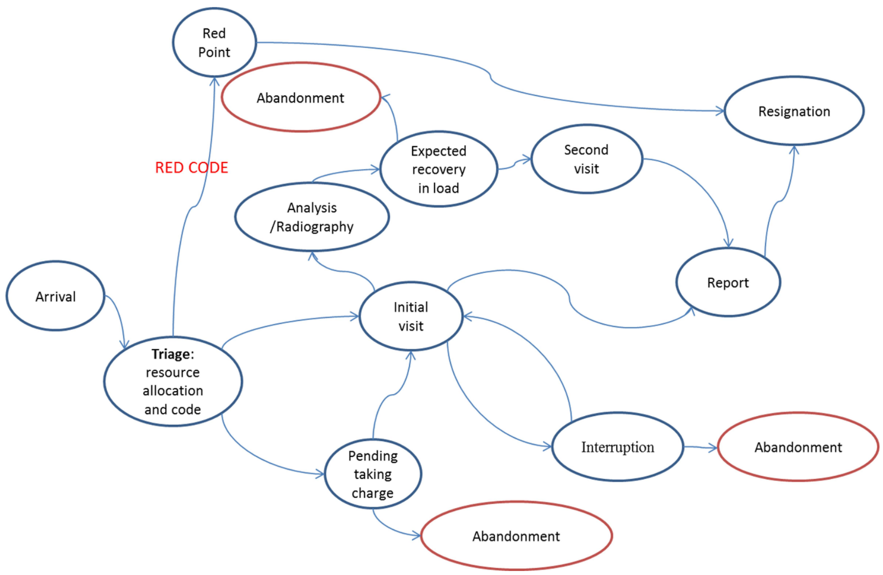


Figure 2. Aid process Markov chain.

Therefore, no transition probabilities are to be deduced from the data concerning generic ED arrival rates, according to the code, the medical resources, and the number of patients undergoing clinical testing. Thus, this chain only defines the states affecting the dynamics of the system-patient; the model focuses on the risk of patient self-discharge from the ED, so the states related to this event are highlighted in red. The model shows the structure and architecture of the chain; it is realistic, because this type of model can be used without regard to any emergency department and has universal value.

Therefore, the difference between distinct health agencies falls within transition probabilities among the states. The states of self-discharge are represented with different colors to highlight the focus of the analysis to be presented later. The chain is reset with the arrival, and it may cover any of the 12 cases previously listed, which can be distinguished by the priority code and the medical resources.

The output event from the chain can be represented by both the abandonment and the discharge from the hospital. The assumptions underlying the model are that:

1. It is a unique model that varies from hospital to hospital, depending on the transition probability to be placed on the arches;
2. The odds on the arches are obtained by the dynamics of entry and resource usage rates;
3. The model is valid just for the priority code.

The Markov chain behavioral

Once shown the generic model of finite automaton transition, probabilities must be included on the arches abutting states. Before describing how to affix values that can be derived from strings, an additional model of Markov's behavioral chain has to be shown, highlighting how the patient's satisfaction can affect the voluntary departure from the ED. The following analysis is based on the assumption that the patient's self-discharge is the result of a high level of dissatisfaction and discontent, mainly due to long waits. When passing from one state to another, the patient can be satisfied, not very satisfied, or dissatisfied. In the last case, the possible result is self-discharge; in the first case (patient satisfied), he or she will go to the next State, likely without complaint. Poor satisfaction creates a greater likelihood of dissatisfaction (and then transition to the state of self-discharge) in the following stages. Markov's chain described here is then complicated by the presence of states representing the degree of satisfaction. The description of the following chain in Figure 3 reflects what has been previously described.

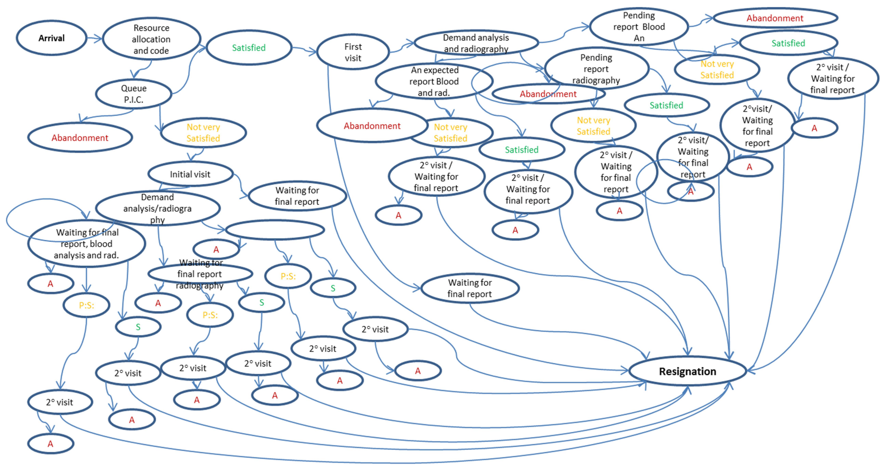


Figure 3. Behavioral Markov chain.

You can also distinguish additional states of satisfaction. For example, following assignment of the priority code and medical resource, the patient, depending on the time spent waiting, can change his state of satisfaction and pass to the next state (pending Acceptance). This means that you can reach three states: satisfied, not very satisfied, self-discharge (or dissatisfaction). Starting from the first two, two identical states are open, two identical but with different transition probabilities in terms of customer satisfaction. This representation derives from the consideration that if, despite the long waiting times, the patient is willing to wait further, his or her tendency to self-discharge will increase according the following phases. This supposes that the transition probabilities toward the state of discharge will be higher if that part of the chain originates from a state of little satisfaction. To simplify the representation, at the end of the chain, the state of self-discharge is indicated with a red status and low satisfaction with the ED in yellow. In addition, from the second state or waiting time, the outcome does not appear to be a very satisfied state because the patient is supposed to be at the end of the process, waiting until discharge or leaving.

For each of the 12 cases, the transition probabilities will change because each case will feature different priorities, different availability of resources, and, consequently, different waiting times.

The chain before the one just described in Figure 4, should also be considered.

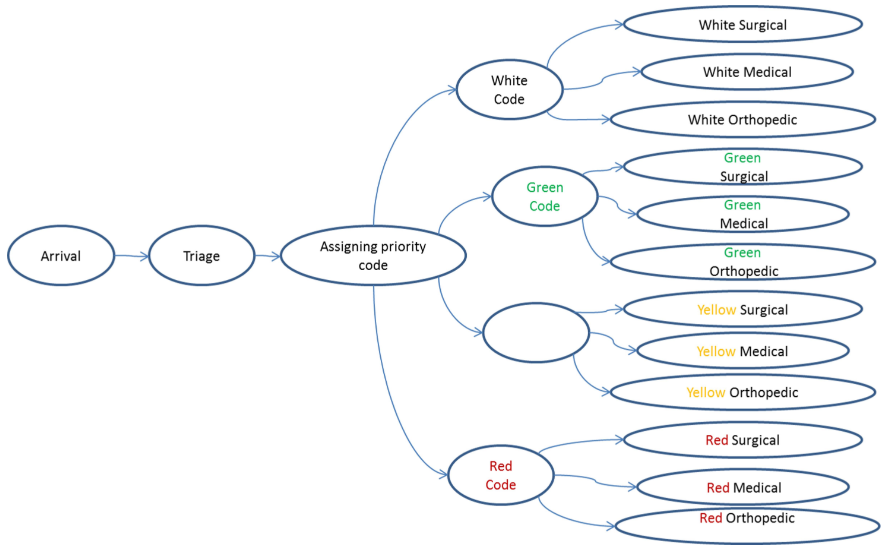


Figure 4. Resource assignment and chain code.

Here, we highlight the transitions between arrival and code assignment and then the resource assignment; starting from each of the 12 states, the behavioral chain will start as shown in Figure 3.

It is now possible to describe the transition probabilities. To compute these probabilities, it is necessary to obtain aid job data, arrival rates, waiting time, and the average number of emergency department patients who need blood and/or radiographic analysis.

We can distinguish between two types of transition probabilities:

- the odds that derive from the arrival rates and the average number of patients who require further examinations;
- the odds that derive from waiting time and consequently define the transition toward a State of satisfaction.

For the first type, we assumed we had arrival rates corresponding to the transition probability; for example, in Figure 4 for the transition probability from a code assignment state to a priority code state, we considered the percentage of patients that (during this period) were assigned that particular priority code. The same assumption is proposed for the medical resource assignment and for transitions depending on analysis requests and waiting times for analysis/radiography results (considered the percentage of patients who were assigned a particular code and of a particular medical resource for which specific analyses were requested).

Establishing the second type of probability is more complex because it consists of transforming a satisfaction index in a transition probability. The chosen approach is derived from two considerations:

- the available data is the patient's waiting time in the ED;
- dissatisfaction is generated by waiting times that are too long.

It is then considered a normal distribution, where the mean corresponds to the sample mean and the variance to the variance. Based on this distribution, the limitations in terms of satisfaction are a lower bound to represent the maximum wait time because a patient is satisfied and an upper bound that represents the maximum wait time beyond which the patient feels unsatisfied and is led to self-discharge. Between the two limits is an area that corresponds to low satisfaction, in which the patient still waits but with a greater likelihood of self-discharge at the following stages. Defined by these limits, the transition probabilities for behavioral are computed by evaluating the area under the curve, bounded by the limit values for satisfaction.

In Figure 5, they are tracked on the left and on the right boundary (blue lines) of the sample mean (red line).

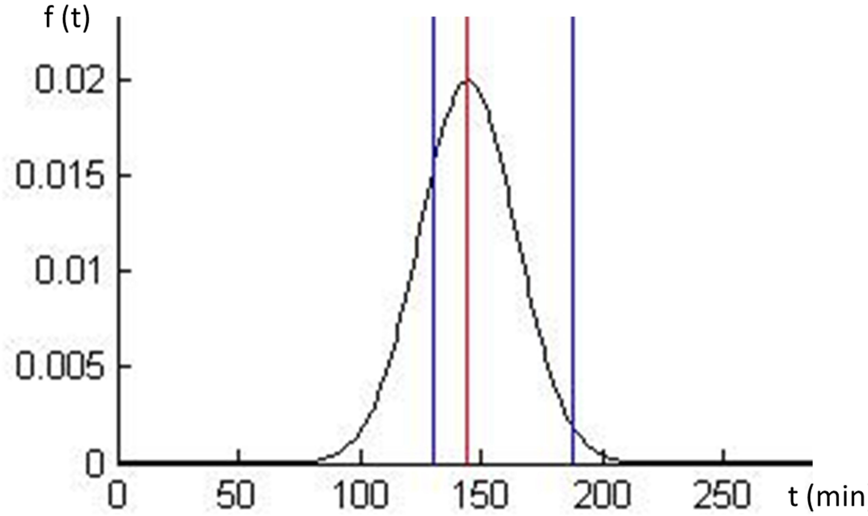


Figure 5. Laboratory report waiting time normal distribution.

MODEL VALIDATION: CASE STUDY

In 120 questionnaires collected, only those that relate to medical cases were considered, and then the limits are defined as hours of satisfaction for each phase and for each code. The questionnaires relating to medical cases amounted to 75, of which 23 were white codes, 32 were green codes, and 20 were yellow codes.

The limit hours of satisfaction are highlighted as follows: a lower limit before which the patient feels satisfied and an upper limit after which the patient declares himself dissatisfied and leaves the emergency department. These limits were deducted from the questionnaires that were completed by waiting patients, using an arithmetic mean of data collected; "wait times adjusted" refers to the lower bounds, whereas "maximum wait times for customer satisfaction" refers to the upper limit.

A distribution of the patient satisfaction in the case of medical green code is shown in Figure 6, which report the limits of the patient's satisfaction in terms of waiting times from the arrival to the emergency room to the admission procedure. More specifically, if the waiting time is below the lower limit, the patient is satisfied; whereas, if the waiting time is higher than the upper limit, the patient is unsatisfied and leaves the emergency room without being seen.

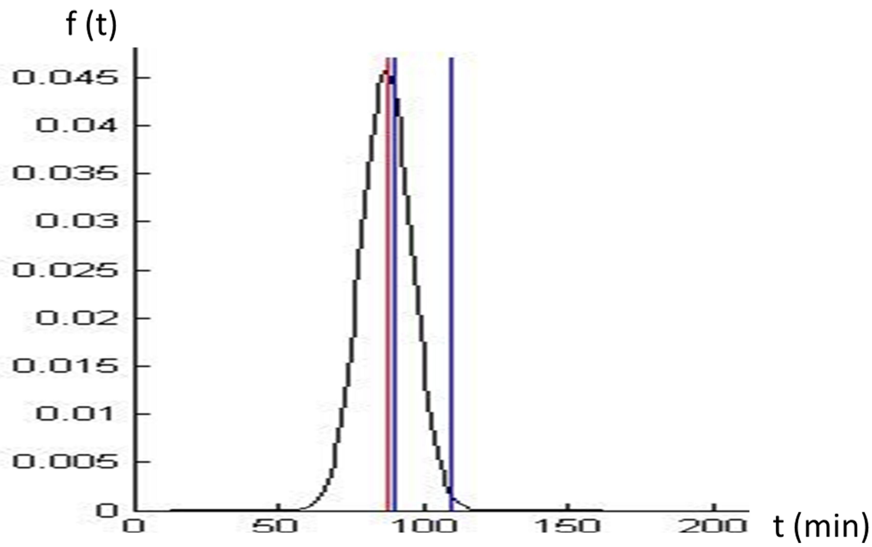


Figure 6. Waiting time distribution.

In Figure 6, the three vertical lines represent the average waiting time before the admission of a green-code patient (red line) and the upper and lower limits, in terms of hours of waiting time, for the satisfaction of patients (blue lines).

In each specific transition state, the probability of having a patient satisfied ($\mathrm{Pr}_{\mathrm{Sat}}$), not very satisfied ($\mathrm{Pr}_{\mathrm{notSat}}$), or dissatisfied ($\mathrm{Pr}_{\mathrm{Dissat}}$), is equal to the underlying curve areas of the distribution curve, bounded by minimum and maximum waiting times and is computed by using MatLab software:

- $\mathrm{Pr}_{\mathrm{Sat}} = 0.62$ (area on the left of the lower limit)
- $\mathrm{Pr}_{\mathrm{notSat}} = 0.375$ (area between the lower and the upper limits)
- $\mathrm{Pr}_{\mathrm{Dissat}} = 0.005$ (area on the right of the upper limit).

Transition probability chains that represent the three cases (white, green and yellow code) are presented in Figure 7 (white code), Figure 8 (green code) and Figure 9 (yellow code) respectively. For each figure, transition probabilities are reported in bold next to each arrow connecting different states of the process, from the patient arrival to the final discharge.

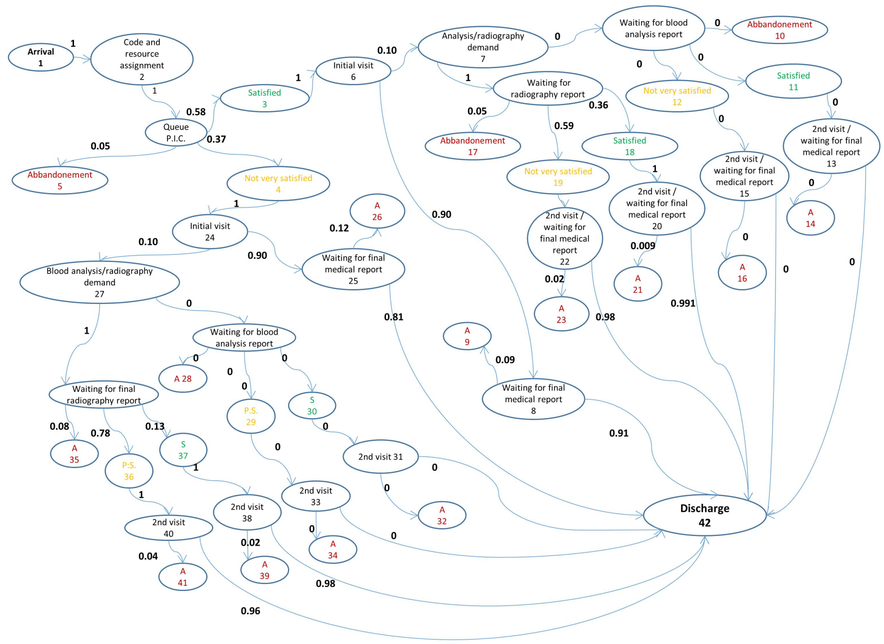


Figure 7. Probability chain: white code.

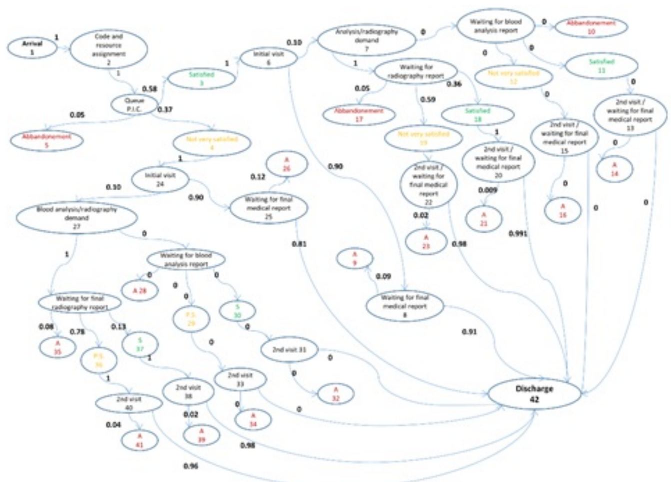


Figure 8. Probability chain: green code.

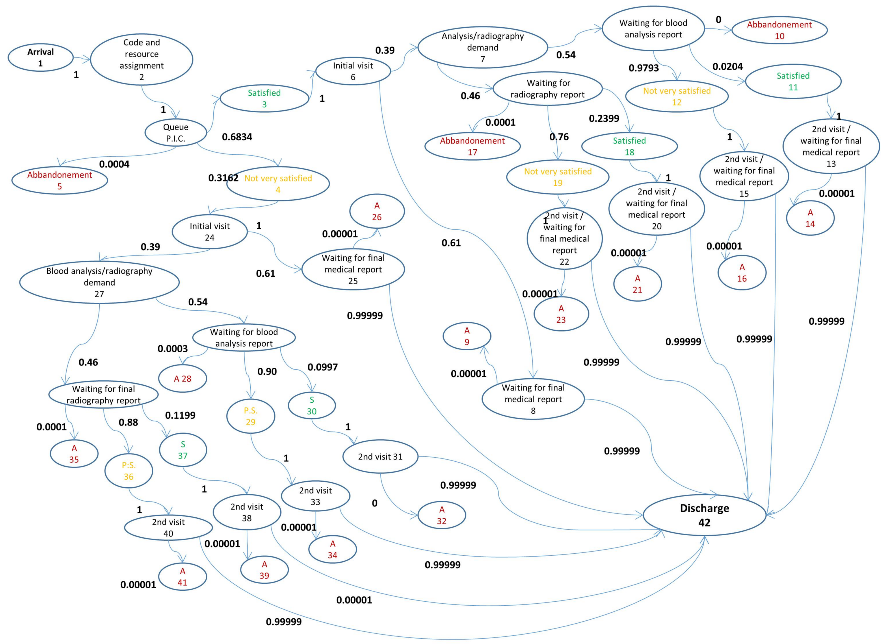


Figure 9. Probability chain: yellow code.

Some peculiarities can be highlighted in the figures:

- Since the white code does not undergoes laboratory analysis, then all the transitions following from "waiting for blood test report" have a probability equal to zero;
- For the yellow code, the probability of dissatisfaction that leads to abandonment is particularly low; this is because the higher-gravity of the condition make the patient more likely to wait even longer to be treated. Nevertheless, the yellow code patients are almost always not very satisfied with the waiting times until discharge.

Chain behavior under current conditions

In order to compute the probability of a patient voluntarily departing from the ED without being seen by a doctor, we calculate the transition probabilities of the chains at nominal conditions by setting the arrival of the patient as initial condition. The sum of the computed probabilities gives the rate of removal from the emergency department throughout the profiled process.

- For the white code the voluntary departure rate white code is 2.88%.
- For the green code the voluntary green code removal rate is 1.35%.
- And, finally, for the yellow code the voluntary departure rate yellow code is 0.0009%.

As expected, the likelihood of abandonment is higher for lower-priority codes, and the yellow code is basically nothing because yellow-code patients are not willing to wait a very long time just to be seen. The number of patients who are at risk of self-discharge during the period in question is obtained by translating these probabilities as follows: From a total of 31 white code patients, one patient is at risk of self-discharge; from a total of about 4,073 green-code patients, 55 patients are at risk of self-discharge. A significant impact on the abandonment rate is generated by the likelihood of self-discharge due to wait time for the final report. In general, the interviewed patient expects the time of discharge to be lower than the average of the times; for the yellow code, high dissatisfaction is caused by waiting for the outcome of laboratory tests.

Sensitivity analysis

Once the model was validated and its operation observed in the present conditions, we wanted to see how they can change the percentages of self-discharge. Of the three cases presented, the case of the code-yellow doctor has a chance of dropping remarkably low; we would prefer then to evaluate the positive changes that can be achieved in cases of lower priority. In particular, we wanted to analyze the impact on the likelihood of self-discharge of a decrease in the average time taken to discharge and the average time taken to obtain the outcome of the analysis.

Medical White code: Decrease in discharge time

For the white code, the sample mean discharge time provides a value of 40.4 minutes, if a decrease of 10% in discharge times means that you experience a mean wait time of 36.4 minutes. Whereas, if the chain starts from "Waiting time for admission" satisfied, the stated limits are 45 minutes to 60 minutes for satisfaction and dissatisfaction, respectively, so you face the previous transition probabilities (0.09 for dropping and 0.81 for discharge):

- $\mathrm{Pr}_{\mathrm{Sat}} = 0.7849$;
- $\mathrm{Pr}_{\mathrm{notSat}} = 0.0152$;
- $\mathrm{Pr}_{\mathrm{Dissat}} = 0.1999$.

If the chain starts from "Waiting time for admission not very satisfied," the stated limits are 35 minutes for satisfaction and 55 for self-discharge. We obtain:

- $\mathrm{Pr}_{\mathrm{Sat}} = 0.4489$;
- $\mathrm{Pr}_{\mathrm{notSat}} = 0.0440$;
- $\mathrm{Pr}_{\mathrm{Dissat}} = 0.5071$.

Simulating the chain again with new transition probabilities for 650 steps, we had a new deployment limit by which the sum of the probabilities of transitions toward self-discharging provides the value of 1.6%. Then, we achieved a good reduction in the rate of voluntary departure, demonstrating the great importance that the transition has had in dropping by waiting for the final report.

Medical White code: Decrease in radiographic reporting times

For the white code, the sample mean of the times reporting of radiographic analysis provides a value of 97.2 minutes, assuming that a decrease of 10% in time means that you experience an average wait time of 87.5 minutes. Whereas, if the chain starts from "Waiting time for admission" satisfied, the stated limits are 90 minutes to 130 minutes for satisfaction and dissatisfaction, so we obtained:

- $\mathrm{Pr}_{\mathrm{Sat}} = 0.5497$;
- $\mathrm{Pr}_{\mathrm{notSat}} = 0.0168$;
- $\mathrm{Pr}_{\mathrm{Dissat}} = 0.4335$.

If the chain starts "Waiting time for admission not very satisfied," the stated limits are 75 min for satisfaction and 125 minutes for self-discharge. We obtained:

- $\mathrm{Pr}_{\mathrm{Sat}} = 0.2660$;
- $\mathrm{Pr}_{\mathrm{notSat}} = 0.0304$;
- $\mathrm{Pr}_{\mathrm{Dissat}} = 0.7036$.

Simulating the chain again with new transition probabilities for 650 steps, we had a new deployment limit by which the sum of the probabilities of transitions toward self-discharging becomes 2.7%. The decrease in the dropout rate is not very high; the motivation lies in the willingness of the patient to wait longer to receive the results of tests.

Medical Green code: Decrease in discharge time

For the green code sample, mean discharge time provides a value of 70.45 minutes. Assuming a decrease in the times of 10% means, we had an average wait time of 63.4 minutes, whereas if the chain starts from "

Waiting time for admission" satisfied, the stated limits are 60 minutes for satisfaction and 100 minutes for dissatisfaction; in the face of previous transition probabilities (0.075 for dropping and 0.925 for discharge), we obtained:

- $\mathrm{Pr}_{\mathrm{Sa\ t}}$= 0.4338;
- $\mathrm{Pr}_{\mathrm{notSat}}$ = 0.0364;
- $\mathrm{Pr}_{\mathrm{Dissat}}$ = 0.5298.

If the chain starts "Waiting time for admission not very satisfied," the stated limits are 50 minutes for satisfaction and 97.5 for dropping. In the face of previous transition probabilities, we obtained (0.09 for dropping and 0.91 for discharge):

- $\mathrm{Pr}_{\mathrm{Sat}}$ = 0.2556;
- $\mathrm{Pr}_{\mathrm{notSat}}$ = 0.0473;
- $\mathrm{Pr}_{\mathrm{Dissat}}$ = 0.6971.

Simulating the chain again with new transition probabilities for 650 steps, we had a new deployment limit by which the sum of the probabilities of transitions toward abandoning yielded the value of 0.999347%. Then, we obtained a decrease in the rate of voluntary departure, similar to the white-code priority.

Medical Green-code: Decrease report turnaround time of laboratory analysis

For the green-code, the sample mean of laboratory reporting times provides a value of 160.8 minutes. Assuming a decrease in the time of 10% means, we achieved an average wait time of 144.72 minutes, whereas, if the chain starts from " Waiting time for admission" satisfied, the stated limits are 150 minutes for satisfaction and 200 minutes for dissatisfaction, we obtained:

- $\mathrm{Pr}_{\mathrm{Sat}}$ = 0.6041;
- $\mathrm{Pr}_{\mathrm{notSat}}$ = 0.0029;
- $\mathrm{Pr}_{\mathrm{Dissat}}$ = 0.3930.

If the chain starts "Waiting time for admission not very satisfied," the stated limits are 130 min for satisfaction and 188 for dropping. We obtained:

- $\mathrm{Pr}_{\mathrm{Sat}}$ = 0.2309;
- $\mathrm{Pr}_{\mathrm{notSat}}$ = 0.0152;
- $\mathrm{Pr}_{\mathrm{Dissat}}$= 0.7539.

Simulating the chain again with new transition probabilities for 650 steps, we achieved a new deployment limit in which the sum of the probabilities of transitions toward self-discharging provided the value of 1.34%. The decrease in the dropout rate is not very high; the motivation is to be found, as in the case of the white code, in the will of the patient to wait longer to receive the results of the tests. Nevertheless, the level of dissatisfaction is lower, indicating an opportunity to improve the perception of quality of the department in the view of the patient.

In conclusion, it can be stated that most of the risk of self-discharge is during the wait time before the patient is seen and the wait time for the final test report; generally, once the analysis (which are radiographic or from the laboratory) is requested, the patient, although not very satisfied, is willing to wait longer for the results. To operate continuous quality improvement activities, it is necessary to eliminate waste throughout the process and thus reduce the time required to carry out individual tasks and, therefore, the patient wait time. In this way, as evidenced by the sensitivity analysis, you would be able to reduce the level of dissatisfaction and, consequently, the rate of voluntary departure from the ED.

DISCUSSION AND CONCLUSION

This paper describes a new methodological approach to describe and predict the probability of patients' voluntary departure from EDs. In particular, the Markov Chain Theory has been applied to the first aid process (Figure 1) and modified in order to take into account the patient's satisfaction. Therefore, a Behavioral Markov Chain (Figure 3) model has been developed, compared to the Markov Chain one (Figure 2), and adopted as a novel strategy to model the ED from a patient's perspective and to offer a potential way to predict the probability of self-discharge of patients from ED.

In the behavioral chain model, the states represent level of patient's satisfaction and can be achieved with the transition probabilities that depend on waiting times. A method to calculate transition probabilities related to patients' satisfaction is proposed (Figure 5).

The proposed model is then applied and validated to a real case study (Figure 6-9).

The work tackles an issue of great interest in the field of management and analysis of the processes; it focused on the process that takes place in the ED, whose organization and proper functioning are fundamental to every hospital, because this department is, in most cases, the first contact between the patient and his or her health care. In literature, there are a remarkable number of articles that deal with the topic of patients who leave the hospital before being discharged; here, several organizational solutions are proposed and/or adopted. The adoption of a patient-tracking system, in particular, systems such as RFID and the electronic bracelet, turned out to be the most frequently preferred.

In this paper, the problem of patients discharging themselves against medical advice is analyzed as a risk that the researchers wanted to examine as a model with the help of Markov's theory, one of the most innovative approaches in health care. To our knowledge, indeed, the application of MC to health care processes is not present in literature.

The work aims at describing and validating a new method to calculate the rate of patient self-discharges from the ED, obtained by presenting a generic model, describing the process of first aid using Markov chains, and taking into account the patients' satisfaction.

It is necessary to identify the conditions of patients' satisfaction and then calculate the relative transition probabilities, depending on their expectations. Therefore, the first result is the implementation of a behavioral Markov chain to describe the aid process, in which states form the aid process as seen from the perspective of the patients. The model is presented as generic and thereby adaptable to each healthy facility by changing only the transition probabilities between states. In the final analysis, the probability of patient self-discharge from a hospital "A. Cardarelli" of Naples is calculated according to Markov's theory or by calculating the limit distribution. The abandonment rate corresponds to the result obtained by the sum of the transition probabilities referring to the state of waiver at different phases of the process. This allowed us to carry out an analysis of the impact of each activity and the wait time of each stage, based on the probability calculation. The abandonment rate has been evaluated only for the emergency categories, shown as white, green, and yellow codes, for medical resource use only; it is given then a sensitivity analysis presenting improvement cases. The calculation of the abandonment rate may also be extended to other cases, and, if additional data are available, it will be possible to present a more detailed level than the one proposed in the present model.

NOVELTY OF THE STUDY

The novelty of the here proposed work consists in a new way to apply the Markov Chain Theory to a healthcare process, which is based on the patient perspective and takes into consideration the patients' satisfaction. In particular, the Behavioral Markov Chain is developed and proposed as a tool to model the first aid process and predict the probability of voluntary departures of patients from ED.

LIMITATIONS OF THE STUDY

Even though the developed and presented model is a promising tool to model ED's flow chart and calculate the probability of patients' self-discharge from ED, a validation step needs to be implemented to test the validity of the model on real data.

Conflict of interests

The authors declare no potential conflict of interest

REFERENCES

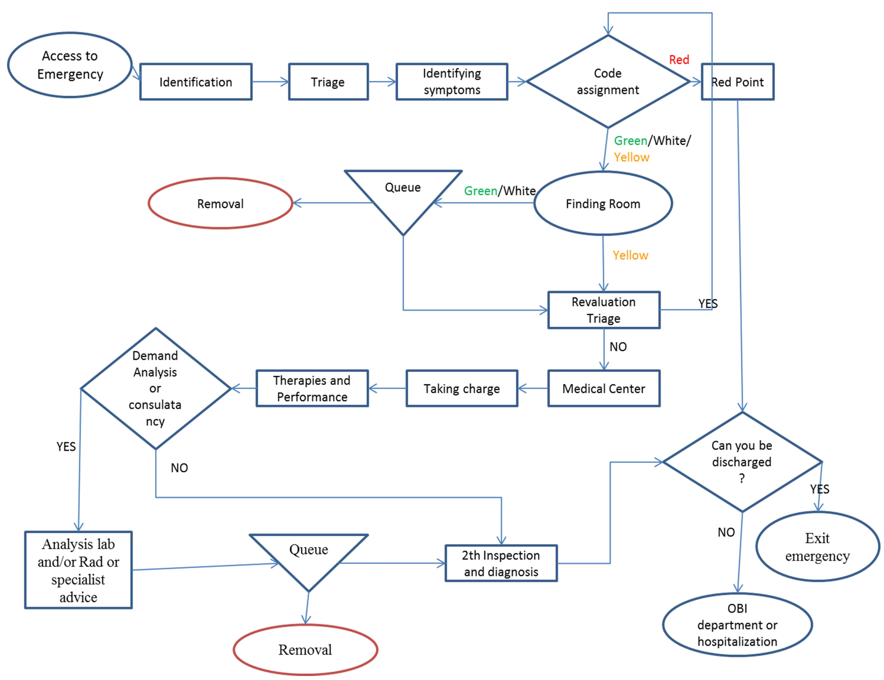
1. Converso G, De Carlini R, Santillo LC, Improta G (2012) Project Management implementation for healthcare activities organization Advances in Computer Science 8:436-443
2. McNulty, T., & Ferlie, E. (2002). Reengineering health care: the complexities of organizational transformation. OUP Oxford.
3. Al-Araidah O., Boran A. and Wahsheh A., Reducing delay in healthcare delivery at outpatients clinics using discrete event simulation, International Journal of Simulation Modelling 11(4) (2012) 185-195.
4. Carmen R., Defraeye M. and Van Nieuwenhuyse I., A decision support system for capacity planning in emergency departments, International Journal of Simulation Modelling 14(2) (2015) 299-312.
5. Mark S. S, Craig F. F (1998) The next generation emergency department. Annals of emergency medicine, 32(1), 65-74.
6. Radnor, Z. J., Holweg, M., & Waring, J. (2012). Lean in healthcare: the unfilled promise?. Social science & medicine, 74(3), 364-371.
7. Affleck A., Parks P., Drummond A., Rowe B. H. and Ovens H. J., Emergency department overcrowding and access block, Canadian Journal of Emergency Medicine 15(6) (2013) 359-370.
8. FitzGerald G., Jelinek G. A., Scott D. and Gerdtz M. F., Republished paper: Emergency department triage revisited, Postgrad. Med. J. 86(1018) (2010) 502-508.
9. Oredsson S., Jonsson H., Rognes J., Lind L., Göransson K. E., Ehrenberg A., Asplund K., Castrén M. and Farrohknia N., A systematic review of triage-related interventions to improve patient flow in emergency departments, Scand. J. Trauma Resusc. Emerg. Med. 19(1) (2011) 43.
10. Storrow A. B., Lindsell C. J., Collins S. P., Fermann G. J., Blomkalns A. L., Williams J. M., Goldsmith B. and Gibler W. B., Emergency department multimarker point-of-care testing reduces time to cardiac marker results without loss of diagnostic accuracy, Point of Care 5(3) (2006) 132-136.
11. Masso M., Bezzina A. J., Siminski P., Middleton R. and Eagar K., Why patients attend emergency departments for conditions potentially appropriate for primary care: reasons given by patients and clinicians differ, Emerg. Med. Australas. 19(4) (2007) 333-340.
12. Weng S.J., Cheng B.-C., Kwong S. T., Wang L.-M. and Chang C.-Y. Proceedings of the Winter Simulation Conference, Winter Simulation Conference: 2011; pp 1231-1238.
13. Abo-Hamad W. and Arisha A., Simulation-based framework to improve patient experience in an emergency department, European Journal of Operational Research 224(1) (2013) 154-166.
14. Di Leva A. and Sulis E., Process analysis for a hospital Emergency Department, International Journal of Economics and Management Systems 2 (2017) 34-41.
15. Zhang X-l, Zhu T, Luo L, He C-z, Cao Y, Shi Y-k Forecasting emergency department patient flow using Markov chain. In: Service Systems and Service Management (ICSSSM), 2013 10th International Conference on, 2013. IEEE, pp 278-282
16. Zhu T, Zhang X-l, Luo L, Shi Y-k, Cao Y Analysis of Patient Flow in Emergency Department Based on Markov Chain. In: The 19th International Conference on Industrial Engineering and Engineering Management, 2013. Springer, pp 829-836

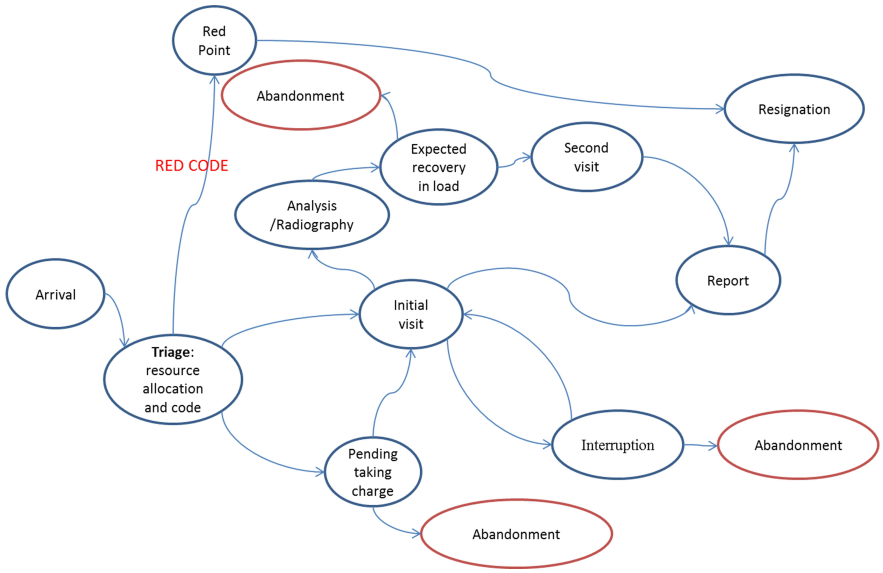
17. Improta G et al. (2017) Improving performances of the knee replacement surgery process by applying DMAIC principles J Eval Clin Pract 23:1401-1407 doi:https://doi.org/10.1111/jep.12810
18. Improta G, Cesarelli M, Montuori P, Santillo LC, Triassi M (2018a) Reducing the risk of healthcare-associated infections through Lean Six Sigma: The case of the medicine areas at the Federico II University Hospital in Naples (Italy) J Eval Clin Pract 24:338-346
19. Improta, G., Balato, G., Ricciardi, C., Russo, M., Santalucia, I., Triassi, M. and Cesarelli, M., "Lean Six Sigma in healthcare", The TQM Journal, 2019a, 31.4: 526-540. https://doi.org/10.1108/TQM-10-2018-0142
20. Improta G., Ricciardi C., Borrelli A. et al. "The application of six sigma to reduce the pre-operative length of hospital stay at the hospital Antonio Cardarelli", International Journal of Lean Six Sigma, 2019b, https://doi.org/10.1108/IJLSS-02-2019-0014
21. Montella E, Di Cicco MV, Ferraro A, Centobelli P, Raiola E, Triassi M, Improta G (2017) The application of Lean Six Sigma methodology to reduce the risk of healthcare–associated infections in surgery departments J Eval Clin Pract 23:530-539
22. Ricciardi, C., Fiorillo, A., Valente, A., Borrelli, A., Verdoliva, C., Triassi, M. and Improta, G. (2019), "Lean Six Sigma approach to reduce LOS through a diagnostic-therapeutic-assistance path at A.O.R.N. A. Cardarelli", The TQM Journal, Vol. 31 No. 5, pp. 657-672.*https://doi.org/10.1108/TQM-02-2019-0065*
23. Improta G, Russo MA, Triassi M, Converso G, Murino T, Santillo LC (2018b) Use of the AHP methodology in system dynamics: Modelling and simulation for health technology assessments to determine the correct prosthesis choice for hernia diseases Math Biosci 299:19-27
24. Improta, G., Romano, M., Di Cicco, M. V. et al. Lean thinking to improve emergency department throughput at AORN Cardarelli hospital. BMC health services research, 2018c, 18.1: 914.
25. Improta, G., Converso, G., Murino, T., Gallo, M., Perrone, A., & Romano, M. (2019c). Analytic Hierarchy Process (AHP) in dynamic configuration as a tool for Health Technology Assessment (HTA): the case of biosensing optoelectronics in Oncology. International Journal of Information Technology & Decision Making (IJITDM), 18(05), 1533-1550.
26. Improta, G., Perrone, A., Russo, M. A., & Triassi, M. (2019d). Health technology assessment (HTA) of optoelectronic biosensors for oncology by analytic hierarchy process (AHP) and Likert scale. BMC medical research methodology, 19(1), 140.
27. Johnson, M., Myers, S., Wineholt, J., Pollack, M., & Kusmiesz, A. L. (2009). Patients who leave the emergency department without being seen. Journal of Emergency Nursing, 35(2), 105-108.
28. Bambi, S., Scarlini, D., Becattini, G., Alocci, P., & Ruggeri, M. (2011). Characteristics of patients who leave the ED triage area without being seen by a doctor: a descriptive study in an urban level II Italian University Hospital. Journal of Emergency Nursing, 37(4), 334-340.
29. Considine J, Kropman M, Kelly E, Winter C (2008) Effect of emergency department fast track on emergency department length of stay: a case–control study Emerg Med J 25:815-819
30. Day TE, Al-Roubaie AR, Goldlust EJ (2012) Decreased length of stay after addition of healthcare provider in emergency department triage: a comparison between computer-simulated and real-world interventions Emerg Med J:emermed-2012-201113
31. Mitchell J, Hayhurst C, Robinson S (2004) Can a senior house officer's time be used more effectively? Emerg Med J 21:545-547
32. Baker D. W., Stevens C. D. and Brook R. H., Patients who leave a public hospital emergency department without being seen by a physician: causes and consequences, Jama 266(8) (1991) 1085-1090.
33. Bazarian J, McClung J, Cheng Y, Flesher W, Schneider S (2005) Emergency department management of mild traumatic brain injury in the USA Emerg Med J 22:473-477
34. Clarey A, Cooke M (2012) Patients who leave emergency departments without being seen: literature review and English data analysis Emerg Med J 29:617-621
35. Amber, R., & Everett, V. B. (1996). Emergency department patient tracking: a cost-effective system using bar code technology. Journal of Emergency Nursing, 22(3), 190-195.
36. Diep T, Kenne J-P, Dao T-M (2010) Feedback optimal control of dynamic stochastic two-machine

flowshop with a finite buffer International Journal of Industrial Engineering Computations 1:95-120

37. Rabbani M, Tanhaie F (2015) A Markov chain analysis of the effectiveness of drum-buffer-rope material flow management in job shop environment International Journal of Industrial Engineering Computations 6:457-468
38. Sonnenberg, F. A., & Beck, J. R. (1993). Markov models in medical decision making: a practical guide. Medical decision making, 13(4), 322-338.
39. Hashmi MF, Hambarde AR, Keskar AG (2014) Robust image authentication based on HMM and SVM classifiers Engineering Letters 22:183-193
40. Reinhard, J. M. (1984). On a class of semi-Markov risk models obtained as classical risk models in a Markovian environment. ASTIN Bulletin: The Journal of the IAA, 14(1), 23-43.
41. Brooks S (2002) Markov chain Monte Carlo method and its application Journal of the Royal Statistical Society: Series D (The Statistician) 47:69-100 doi:10.1111/1467-9884.00117
42. Knorr-Held, L., & Besag, J. (1998). Modelling risk from a disease in time and space. Statistics in medicine, 17(18), 2045-2060.
43. Roy SB, Maria M, Wang T, Ehlers A, Flum D (2018) Predicting Adverse Events After Surgery Big Data Research doi:https://doi.org/10.1016/j.bdr.2018.03.003
44. Ricciardi C., Amboni M., De Santis C. et al. Using gait analysis' parameters to classify Parkinsonism: a data mining approach. Computer Methods and Programs in Biomedicine, 2019, 105033. https://doi.org/10.1016/j.cmpb.2019.105033
45. Ricciardi C., Amboni M., De Santis C. et al. (2020) Classifying Different Stages of Parkinson's Disease Through Random Forests. In: Henriques J., Neves N., de Carvalho P. (eds) XV Mediterranean Conference on Medical and Biological Engineering and Computing – MEDICON 2019. MEDICON 2019. IFMBE Proceedings, vol 76. Springer, Cham
46. Mannarino T., Assante R., Ricciardi C., Zampella E., … and Acampa W. Head-to-head comparison of diagnostic accuracy of stress-only myocardial perfusion imaging with conventional and cadmium-zinc telluride single-photon emission computed tomography in women with suspected coronary artery disease. Journal of Nuclear Cardiology (2019). https://doi.org/10.1007/s12350-019-01789-7
47. Ricciardi C., Cantoni V., Green R., Improta G., Cesarelli M. (2020) Is It Possible to Predict Cardiac Death?. In: Henriques J., Neves N., de Carvalho P. (eds) XV Mediterranean Conference on Medical and Biological Engineering and Computing – MEDICON 2019. MEDICON 2019. IFMBE Proceedings, vol 76. Springer, Cham
48. Ricciardi, C., Cantoni, V., Improta, G., Iuppariello, L., Latessa, I., Cesarelli, M., … & Cuocolo, A. (2020). Application of data mining in a cohort of Italian subjects undergoing myocardial perfusion imaging at an academic medical center. Computer Methods and Programs in Biomedicine, 105343.
49. Valeria R., Ricciardi C., Cuocolo R., Stanzione A., …Maurea S. Machine learning analysis of MRI-derived texture features to predict placenta accreta spectrum in patients with placenta previa, Magnetic Resonance Imaging, 2019, ISSN 0730-725X, https://doi.org/10.1016/j.mri.2019.05.017
50. Ricciardi C., Cuocolo R, Cesarelli G. et al. (2020) Distinguishing Functional from Non-functional Pituitary Macroadenomas with a Machine Learning Analysis. In: Henriques J., Neves N., de Carvalho P. (eds) XV Mediterranean Conference on Medical and Biological Engineering and Computing – MEDICON 2019. MEDICON 2019. IFMBE Proceedings, vol 76. Springer, Cham
51. ROMEO, V., CUOCOLO, R., RICCIARDI, C., UGGA, L., COCOZZA, S., VERDE, F., … & ELEFANTE, A. (2020). Prediction of Tumor Grade and Nodal Status in Oropharyngeal and Oral Cavity Squamous-cell Carcinoma Using a Radiomic Approach. Anticancer Research, 40(1), 271-280.
52. Harms, R. L., & Roebroeck, A. (2018). Robust and fast Monte Carlo Markov Chain sampling of diffusion MRI microstructure models. bioRxiv, 328427.
53. Yuan, Z. (1999). Prediction of protein subcellular locations using Markov chain models. FEBS letters, 451(1), 23-26.
54. Ozkaynak M, Dziadkowiec O, Mistry R, Callahan T, He Z, Deakyne S, Tham E (2015) Characterizing workflow for pediatric asthma patients in emergency departments using electronic health records Journal of biomedical informatics 57:386-398

55. Ben-Assuli O, Leshno M (2013) Using electronic medical records in admission decisions: a cost effectiveness analysis Decision Sciences 44:463-481
56. Helm JE, AhmadBeygi S, Van Oyen MP (2011) Design and analysis of hospital admission control for operational effectiveness Production and Operations Management 20:359-374
57. Prodel M, Augusto V, Xie X Hospitalization admission control of emergency patients using markovian decision processes and discrete event simulation. In: Simulation Conference (WSC), 2014 Winter, 2014. IEEE, pp 1433-1444
58. Lee H-R, Lee T Markov decision process model for patient admission decision at an emergency department under a surge demand Flexible Services and Manufacturing Journal 2018:1-25
59. Lee HK, Li J, Musa AJ, Bain PA A Markov chain model to evaluate patient transitions in small community hospitals. In: 2016 IEEE International Conference on Automation Science and Engineering (CASE), 21-25 Aug. 2016. pp 675-680. doi:10.1109/COASE.2016.7743468
60. Green, L. V., Soares, J., Giglio, J. F., & Green, R. A. (2006). Using queueing theory to increase the effectiveness of emergency department provider staffing. Academic Emergency Medicine, 13(1), 61-68.
61. Wang J., Li J. and Howard P. K., A system model of work flow in the patient room of hospital emergency department, Health care management science 16(4) (2013) 341-351.
62. Improta G., Parente G., Triassi M., Pitingolo G., Cozzolino S. and M. Romano M. C., Evaluation of a Set of indicators devoted to the service of health Education:the case of the biotechnology Centre of A.O.RN. Hospital "A.Cardarelli" in Naples., ICIEMS 2013:15th International Conference on Industrial Engineering and Managment Sciences (2013)




Red Point
Abandonment
Resignation
RED CODE
Expected recovery in load
Second visit
Analysis /Radiography
Arrival
Initial visit
Report
Triage: resource allocation and code
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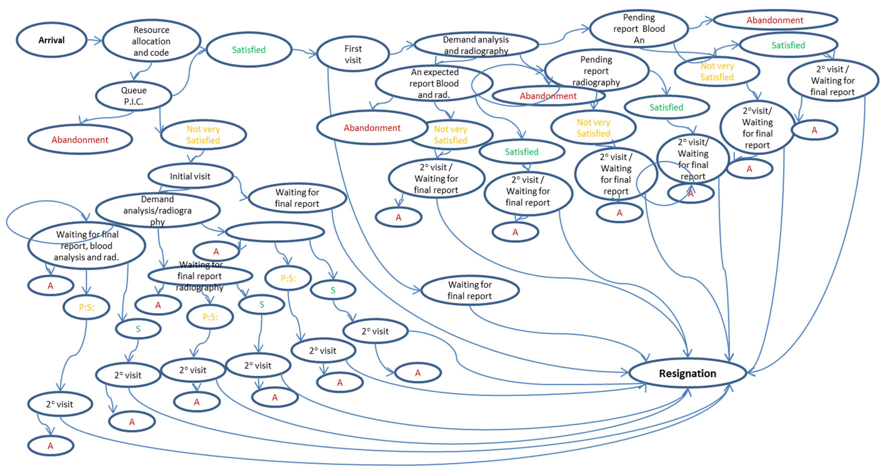
Interruption
Abandonment
Abandonment


Arrival
Resource allocation and code
Satisfied
First visit
Demand analysis and radiography
Pending report Blood An
Abandonment
Satisfied
Queue P.I.C.
An expected report Blood and rad.
Pending report radiography
Not very Satisfied
2° visit / Waiting for final report
Abandonment
Abandonment
Satisfied
Abandonment
Not very Satisfied
Not very Satisfied
Not very Satisfied
Satisfied
2°visit/ Waiting for final report
Initial visit
Waiting for final report
2° visit / Waiting for final report
2° visit / Waiting for final report
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2° visit/ Waiting for final report
Demand analysis/radiography
Waiting for final report, blood analysis and rad.
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Waiting for final report
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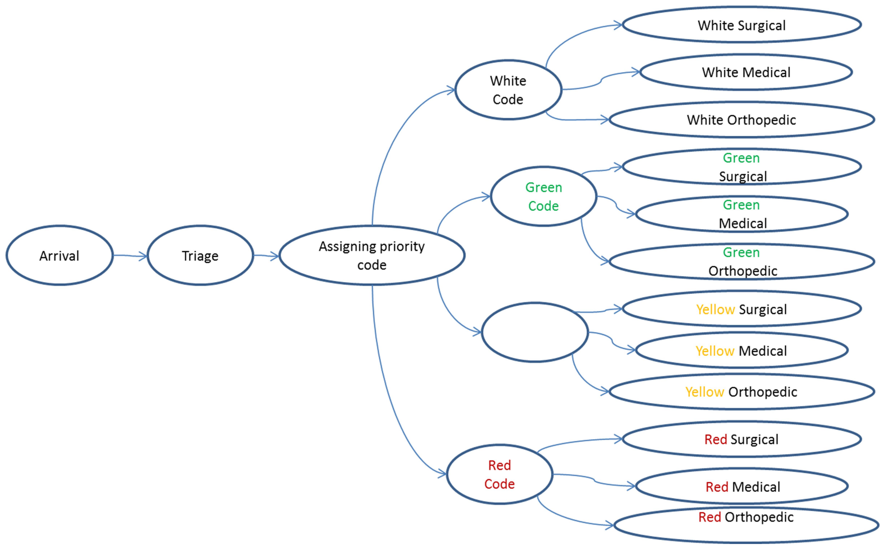
Arrival
Triage
Assigning priority code
White Code
Green Code
Red Code
White Surgical
White Medical
White Orthopedic
Green Surgical
Green Medical
Green Orthopedic
Yellow Surgical
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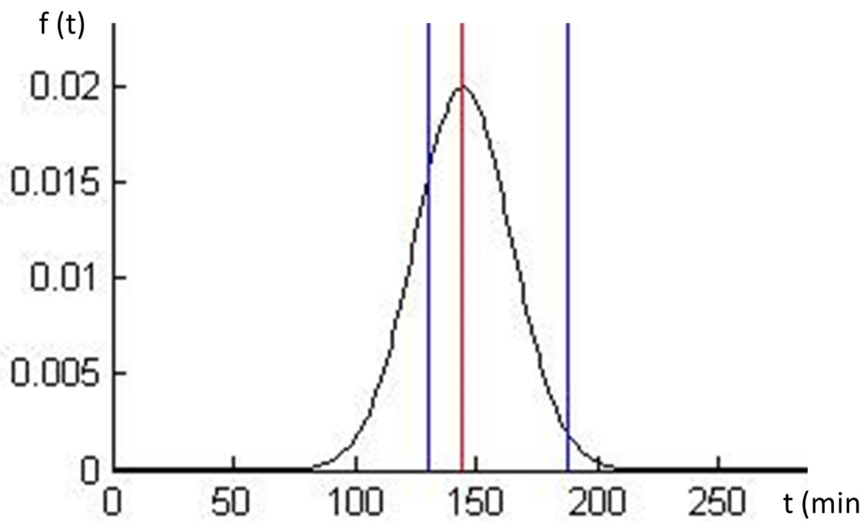
Red Surgical
Red Medical
Red Orthopedic


f (t)
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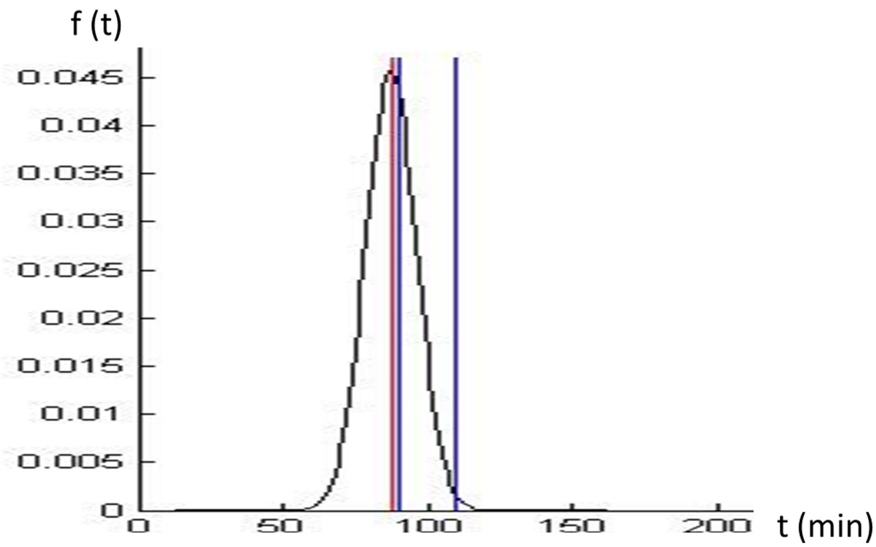
0.015
0.01
0.005
0
0
50
100
150
200
250
t (min)


f (t)
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0.04
0.035
0.03
0.025
0.02
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0.005
0
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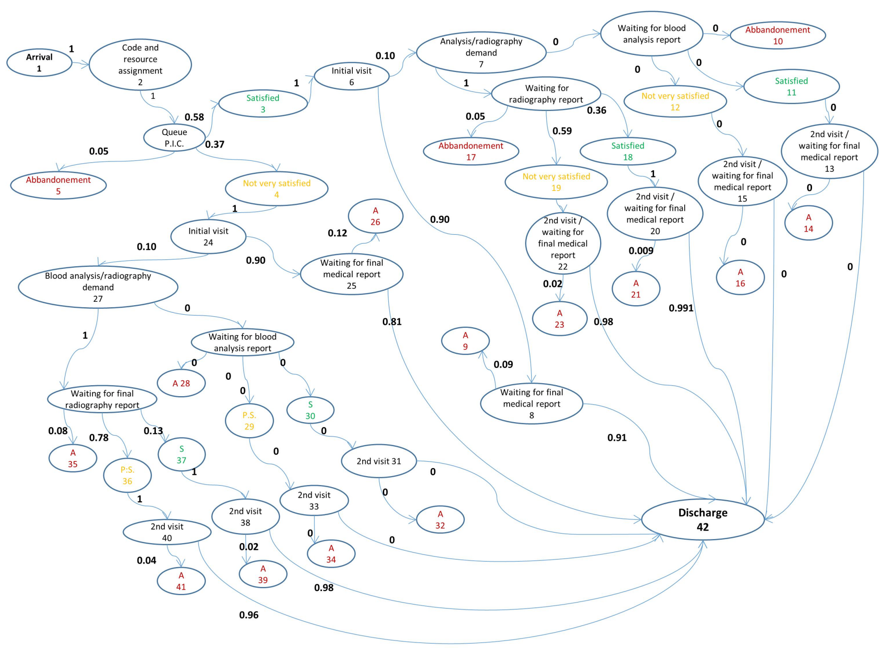
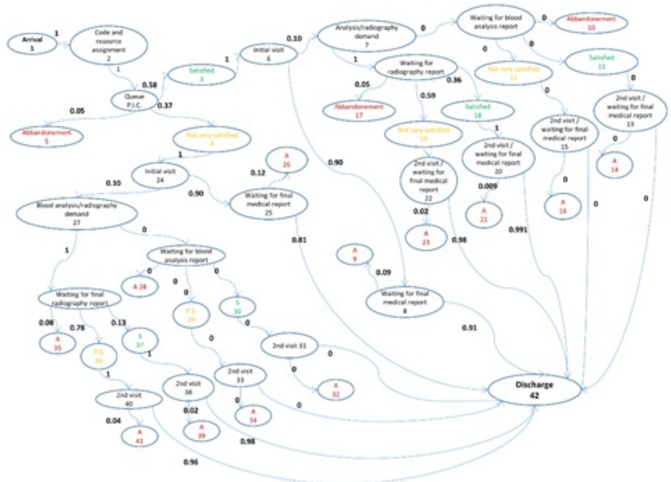
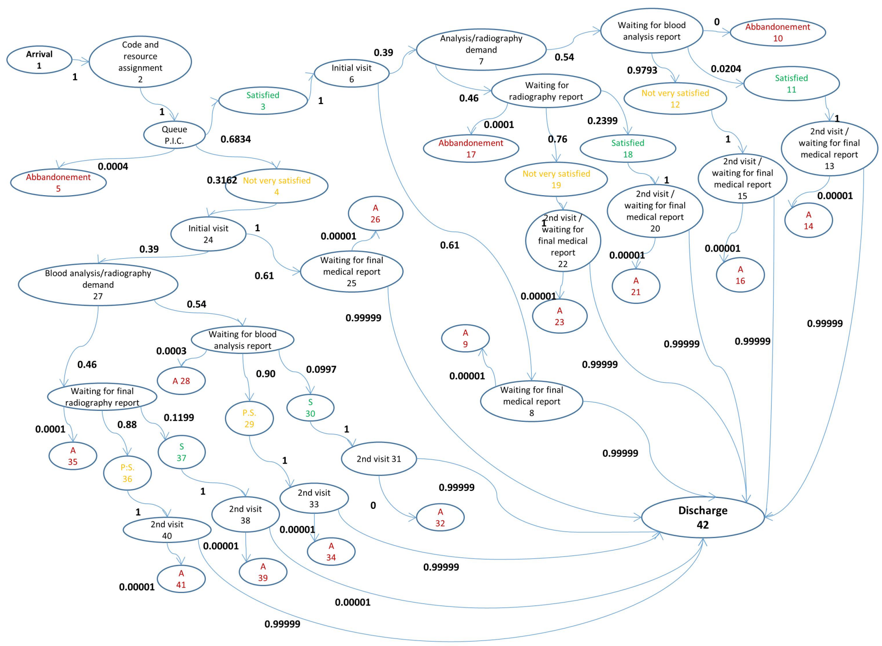
100
150
200
t (min)


Arrival 1
Code and resource assignment 2
Satisfied 3
Not very satisfied 4
Abbandonement 5
Initial visit 6
Analysis/radiography demand 7
Waiting for final medical report 8
A 9
Waiting for blood analysis report
Abbandonement 10
Satisfied 11
Not very satisfied 12
2nd visit / waiting for final medical report 13
Waiting for radiography report
Abbandonement 17
Satisfied 18
Not very satisfied 19
Queue P.I.C.
Initial visit 24
Waiting for final medical report 25
Blood analysis/radiography demand 27
Waiting for final radiography report
2nd visit 31
2nd visit 33
2nd visit 38
2nd visit 40
Discharge 42




Arrival
1
Code and resource assignment
2
Queue P.I.C.
Satisfied
3
Not very satisfied
4
Abbandonement
5
Initial visit
6
Analysis/radiography demand
7
Waiting for final medical report
8
A
9
Waiting for blood analysis report
Abbandonement
10
Satisfied
11
Not very satisfied
12
2nd visit / waiting for final medical report
13
A
14
2nd visit / waiting for final medical report
15
A
16
Abbandonement
17
Satisfied
18
Not very satisfied
19
2nd visit / waiting for final medical report
20
A
21
2nd visit / waiting for final medical report
22
A
23
Initial visit
24
Waiting for final medical report
25
A
26
Blood analysis/radiography demand
27
A 28
P.S.
29
S
30
2nd visit 31
A
32
2nd visit
33
A
34
A
35
P.S.
36
S
37
2nd visit
38
A
39
2nd visit
40
A
41
Discharge
42
0.6834
0.3162
0.0004
0.39
0.54
0.46
0.9793
0.0204
0.0001
0.76
0.2399
0.61
0.00001
0.99999
0.90
0.0997
0.0003
0.88
0.1199
0