

Statistical Merits of U Control Chart when Using VSSI Sampling Policy

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Abstract

Many scientific researches have shown an obvious fact that the quality control charts with variable sampling schemes are more effective than the classical ones in improving statistical measures. The average number of false alarms (ANF), the average number of samples (ANS), the average number of inspected items (ANI), and the adjusted average time to signal (AATS) are the most important statistical measures that have always been attending in the evaluation of control charts. In this paper, a comprehensive analytical review on the U control chart by the statistical measures have been explained. For this purpose, different levels of the possible factors are determined and presented the results of calculating the statistical measures with the obtained parameters on the sampling schemes of the U control chart. It is shown that the variable U control charts are able to improve the effectiveness statistical, especially for detecting shifts and number of false alarms.

Introduction

The basis of variable sampling schemes is that if the drawn sample is close enough to the centerline of the control chart, there is no reason to think that there has been a shift in the process. In this state, the next sample with smaller sample size and/or longer sampling interval drawn on a control chart with more comprehensive control limits. On the other hand, if the current drawn sample is close to the control limits, it is likely that a shift has occurred in the process. Therefore, the next sample with larger sample size and/or shorter sampling interval is drawn on a control chart with tighter control limits to identify the potential shift in the process.

The most common variable sampling schemes, which are also called adaptive schemes, are VSS11Variable sample size, VSI22Variable sampling intervals, and VSSI33Variable sample size and sampling intervals, in which, sample size, sampling interval and sample size with sampling interval are variable, respectively. The VSI sampling scheme presented by Reynolds et al. [1] for the X control chart, they evaluated the performance of this type of sampling scheme from the statistical aspect. In their study, they conducted a comprehensive review between the VSI sampling scheme and the traditional sampling scheme or fix ratio sampling (FSSI) for different shifts in the process mean. Comparison between standard deviation, coefficient of variation, the average number of samples to signal, the average time to signal indicated the acceptable performance of the VSI sampling scheme. Saccucci et al. [2] also investigated the effect of using this scheme compared with FSSI scheme on the performance of the exponentially weighted moving average (EWMA) and the cumulative sum (CUSUM). Prabhu et al. [3] and Costa [4] by presenting a VSS scheme and designing X control chart with two sample sizes, conducted their studies on the impact of this scheme on the measures and performance of the control chart. Similarly, Castagliola et al. [5] by using two statistical measures average sample sizes and truncated average run length, evaluated the X control chart with the VSS sampling

scheme. In more practical applications, Nikolaidis et al. [6] concluded that the use of variable control charts did not have much complexity compared than traditional control charts, but better performance achieved from statistically and economically aspects. Also, Lin and Chou [7] evaluated the VSSI sampling scheme that presented by Prabhu et al. [8] and analyzed the effect of using this sampling method on the X control chart under normality and non-normality conditions. Zhou [9] by considering estimated parameters, studied VSSI scheme for the X control chart and evaluated average time to signal (ATS) in different states. In general, the VSSI sampling scheme was more complex than the VSS and VSI schemes, but it proved to be more efficient in many studies. The researches expressed were just a few cases that examined the statistical performance of sampling schemes for control charts. The researches expressed were just a few cases that examined the statistical performance of sampling schemes for control charts that more information can be found on Khoo et al. [10], Costa and Machado [11], Cheng et al. [12], Lim et al. [13] and Chong et al. [14].

In studies for the U control chart can be noted to Shojaie-Navokh et al. [15] that their research to evaluate the economic-statistical U control chart with variable sampling schemes. In their review by considering the ANF and AATS as constraints and a cost function as the objective function, developed the economic-statistical model and evaluated and compared different sampling methods. Also, in their research, the VSSI scheme was identified as the best scheme with the lowest cost. However, since the main goal of providing variable sampling methods was improvement the statistical measures, in this research, the ANF, ANS, ANI, and AATS measures are reviewing to evaluate the performance of VSSI scheme and other sampling schemes. In the next section, the concept and approach of the Markov chain in designing sampling schemes discussed. In the third section, calculating and introducing statistical measures is explained. The fourth section discussed the evaluation and comparison of statistical sampling schemes under numerical examples and the results presented in the fifth section.

Markov chain approach

Designing variable sampling schemes are based on Markov chain concepts and transition states in a process. This approach expresses that for predicting the action of a system in the future, it is only sufficient to consider the current state of the system. That is, the current state of the process is essential, and the previous states does not have any effect on future states. Therefore, if X_i is a random variable defined in the probability region, then:

$$P[X_{n+1} = j | X_1 = i_1, X_2 = i_2, \dots, X_n = i_n] = P[X_{n+1} = j | X_n = i_n] \quad (1)$$

So, the probability of transition (p_{ij}) in the Markov chain is:

$$P[X_{n+1} = j | X_n = i] = P[X_1 = j | X_0 = i] = p_{ij} \quad (2)$$

Where;

- $1 \leq i, j \leq N \quad 0 \leq p_{ij} \leq 1, i, j \in S$
- $1 \leq i, j \leq N \quad \sum p_{ij} = 1, i, j \in S$

Therefore, the Markov chain for sampling schemes can define by a transfer probability matrix (P), where the element is in row i and column j. The transition probability matrix with n state is expressed as follows:

State

1 amp; 2 amp; ... amp; j amp; ... amp; n

$$P = \begin{matrix} & \begin{matrix} 1 & 2 & \vdots & i & \vdots & n \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ \vdots \\ i \\ \vdots \\ n \end{matrix} & \begin{bmatrix} p_{11} & \text{amp}; p_{12} & \text{amp}; \cdots & \text{amp}; p_{1j} & \text{amp}; \cdots & \text{amp}; p_{1n} \\ p_{21} & \text{amp}; p_{22} & \text{amp}; \cdots & \text{amp}; p_{2j} & \text{amp}; \cdots & \text{amp}; p_{2n} \\ \vdots & \vdots & \text{amp}; \vdots & \text{amp}; \ddots & \text{amp}; \vdots & \text{amp}; \ddots & \text{amp}; \vdots \\ p_{i1} & \text{amp}; p_{i2} & \text{amp}; \cdots & \text{amp}; p_{ij} & \text{amp}; \cdots & \text{amp}; p_{in} \\ \vdots & \vdots & \text{amp}; \vdots & \text{amp}; \ddots & \text{amp}; \vdots & \text{amp}; \ddots & \text{amp}; \vdots \\ p_{n1} & \text{amp}; p_{n2} & \text{amp}; \cdots & \text{amp}; p_{nj} & \text{amp}; \cdots & \text{amp}; p_{nn} \end{bmatrix} \end{matrix}$$

In designing sampling schemes based on Markov chain concepts at each sampling stage according to the process state (in-control or out-of-control) and other states, the process has several transition states and one absorbing state. The method used in this paper to design sampling schemes is based on the researches by Costa [16], Faraz and Moghadam [17], Faraz et al. [18] and Shojaie-Navokh et al. [15], which can refer to them. For example, according to Shojaie-Navokh et al. [15], diagram of different process states in the VSSI sampling scheme, which has five transition states and one absorbing state, can be explained in Fig. 1.

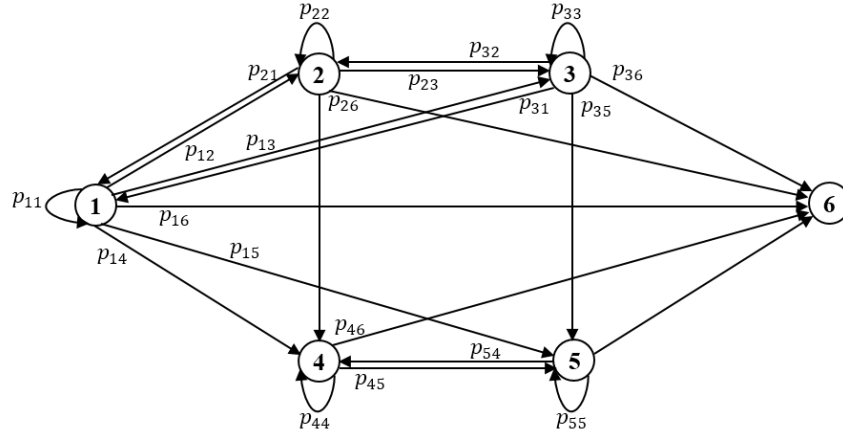


Figure 1 The transition diagram of the VSSI sampling scheme

Statistical measures for performance evaluation of sampling schemes

According to the quality cycle (as shown in Fig. 2) provided by Lorenzen and Vance [19], the process starts from the in-control state (u_0), where occurrence assignable cause leads to a shift in the process mean to out-of-control state (u_+).

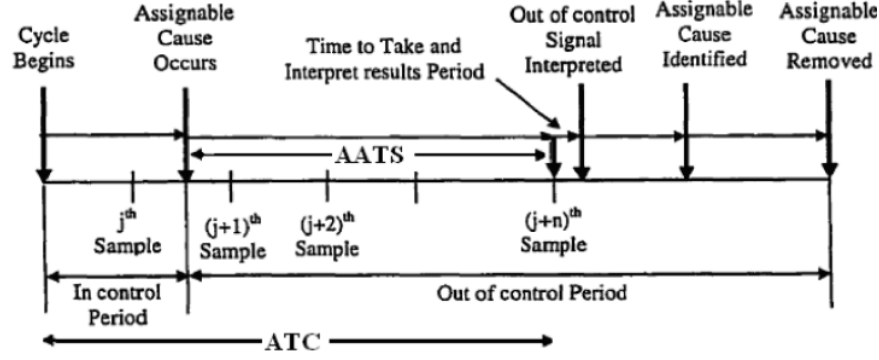


Figure 2 A quality cycle

The average time from the start of the production until the signal by control chart after the process shift is equal to the ATC. This measure can be calculated based on the properties of exponential distribution and Markov chain approach. The expected number of experiments at each stage obtained from the following equation:

$$b(I - Q)^{-1} \quad (3)$$

Where Q is a square matrix by deleting the elements corresponding to the absorbing state of the transition probability matrix P , I is the identity matrix, and b is the initial probabilities vector. Therefore, ATC can be calculated as follows:

$$ATC = b(I - Q)^{-1} h \quad (4)$$

Where h is the vector of interval sampling vector of different process states. Also, assumed that the time before assignable cause occurs is the exponential distribution with parameter λ . Therefore, the average time that the process remains in-control state is λ^{-1} . The mean time from the occurrence of an assignable cause to the time when the control chart detects an out-of-control signal, evaluated by the AATS measure, which equals:

$$AATS = ATC - \lambda^{-1} \quad (5)$$

AATS is the newest measure used to compare the effectiveness of different sampling schemes. AATS shows the control chart sensitivity in detecting shifts in the process. So, with the smaller AATS, the performance of the control chart is better.

The use of variable control charts, moreover reduced AATS, is able to reduce ANS and ANI. The ANS and ANI of different variable control charts are less or equal in comparison with the FSSI sampling scheme. The reduction in measures is also indicated the efficiency of control chart and will have an impact on costs. The values of ANI and ANS for sampling schemes are obtained as follows:

$$ANI = b(I - Q)^{-1} n \quad (6)$$

$$ANS = b(I - Q)^{-1} s \quad (7)$$

Where, n and s are vectors of the number of inspected items and the number of samples, respectively. In designing control charts with variable sampling methods because more focused on AATS, ANS, and ANI, increasing the rate of false alarms is a possibility. Therefore, researchers always have been suspicious to use of these types of control charts. For this purpose, in this paper, we have also compared sampling schemes based on the ANF. Reduction in ANF, as expressed in Fallahnezhad et al. [20], Amiri et al. [21], Faraz and Saniga [22] and Katebi et al. [23] improves the performance of the control chart. The formula for calculating this measure with the vector of false alarms f is similar to other measures based on the Markov chain concepts and is as follows:

$$ANF = b(I - Q)^{-1} f \quad (8)$$

Sensitivity analysis

In this section, the performance of the VSSI, VSS, VSI, and FSSI sampling schemes will be calculated and compared with the ANF, ANS, ANI, and AATS measures for the U control chart by using various numerical examples. The values of b , h , n , s , and f that used to calculate the statistical criteria are based on the references in Section 2, which can be referred for more information. The method used for calculating the control limits (UCL_i), the warning limits (WL_i), and the solution to determine the optimal values of the measures and parameters of the control chart are based on Shojaie-Navokh et al. [15]. In schemes that used two sample sizes (n_1, n_2), the range for choosing values equal; $n_1 = (1, \dots, n_0)$, $n_2 = (n_0, \dots, 50)$ and in schemes that used two sampling intervals (h_1, h_2), the range for choosing values are $h_2 = (0.1, \dots, h_0)$, $h_1 = (h_0, \dots, 8)$. Therefore, after determining the different levels for u_0 , u_+ , λ , n_0 , and h_0 , the results of calculating the statistical measures for the sampling schemes are presented in Tables 1-12 (Tables 2-12 is given in Appendix of this paper). Also, the values of obtained parameters in each table for the VSSI, VSS, VSI, and FSSI sampling schemes respectively are shown as follows: ($n_1, n_2, h_1, h_2, WL_1, WL_2, UCL_1, UCL_2$), ($n_1, n_2, h_0, WL_1, WL_2, UCL_1, UCL_2$), (n_0, h_1, h_2, WL, UCL), (n_0, h_0, UCL).

Table 1 The results of applying the VSSI sampling scheme when $u_+ = 3.2$ and $\lambda = 0.04$

ANS	ANS	ANI	ANI	ANF	ANF	AATS	A
Obtained parameters	Value	Obtained parameters	Value	Obtained parameters	Value	Obtained parameters	Value
(5.04)	108.99	(5.04)	326.96	(5.04)	0.44	(5.04)	8.9
(4.00)	104.06	(4.00)	624.35	(4.00)	1.85	(4.00)	4.0
(3.54)	102.02	(3.54)	918.15	(3.54)	6.53	(3.54)	2.0
(5.04)	33.99	(5.04)	101.97	(5.04)	0.11	(5.04)	17.0
(4.00)	29.06	(4.00)	174.37	(4.00)	0.46	(4.00)	8.0
(3.54)	27.02	(3.54)	243.17	(3.54)	1.61	(3.54)	4.0
(5.04)	15.67	(5.04)	47.00	(5.04)	0.03	(5.04)	27.0
(4.00)	10.74	(4.00)	64.42	(4.00)	0.11	(4.00)	12.0
(3.54)	8.70	(3.54)	78.26	(3.54)	0.41	(3.54)	6.0
(5.04)	11.52	(5.04)	34.56	(5.04)	0.01	(5.04)	36.0
(4.00)	6.59	(4.00)	39.55	(4.00)	0.04	(4.00)	16.0
(3.54)	4.55	(3.54)	40.94	(3.54)	0.13	(3.54)	8.0

According to the obtained results of Tables 1-12, comparing the performance of the proposed sampling schemes by the statistical measures indicates that the VSSI sampling scheme, like Shojaie-Navokh et al. [15] which performed better in reducing costs, here too, has better accuracy and performance in reducing the statistical measures. Although, the VSS and VSI schemes have weaker performance than the VSSI scheme but used them compared to the FSSI scheme reduce statistical measures. The results demonstrate that a small change in the design of U control chart and sampling scheme increases the sensitivity to shifts and performance of the process.

conclusion

The purpose of this paper is to investigate the ANF, ANS, ANI, and AATS statistical measures by applying variable sampling schemes in the U control chart. The U control chart is the statistical process control tools and used to count the number of defects in samples of a product. In designing variable sampling schemes,

the range of sample size and/or sampling interval is unpredictable. Variation in sampling may be unpleasant from an operational viewpoint, but as shown, significant statistical improvement has been generated in the process. In this paper, the design of the sampling schemes and evaluation of statistical measures was done based on the Markov chain approach and transition probability of different process states. Also, by comparing the numerical results, the VSSI sampling scheme, which uses two sample sizes and two sampling intervals, compared with other schemes, has led to more reduction in the statistical measures and increase efficiency in the U control chart. Therefore, the significant advantage of the VSSI sampling scheme with other schemes, especially the FSSI, recommend in practical applications for more accuracy and better results.

CONFLICT OF INTEREST

The authors declare no potential conflict of interest.

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APPENDIX

Table 2 The results of applying the VSSI sampling scheme when $u_0=1.5$ and $\lambda=0.04$

Table 3 The results of applying the VSSI sampling scheme when $u_0=1.5$ and $u_+=3.2$

Table 4 The results of applying the VSS sampling scheme when $u_+=3.2$ and $\lambda=0.04$

Table 5 The results of applying the VSS sampling scheme when $u_0=1.5$ and $\lambda=0.04$

Table 6 The results of applying the VSS sampling scheme when $u_0=1.5$ and $u_+=3.2$

Table 7 The results of applying the VSI sampling scheme when $u_+=3.2$ and $\lambda=0.04$

Table 8 The results of applying the VSI sampling scheme when $u_0=1.5$ and $\lambda=0.04$

Table 9 The results of applying the VSI sampling scheme when $u_0=1.5$ and $u_+=3.2$

Table 10 The results of applying the FSSI sampling scheme when $u_+=3.2$ and $\lambda=0.04$

Table 11 The results of applying the FSSI sampling scheme when $u_0=1.5$ and $\lambda=0.04$

Table 12 The results of applying the FSSI sampling scheme when $u_0=1.5$ and $u_+=3.2$

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