Partially-Averaged Navier-Stokes Equations Model for Prediction of Turbulent Ocean Flows

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Abstract

The accurate simulation of oceanic turbulence is crucial to understanding global ocean circulation, impacting accurate prediction of global warming effects and national security problems. Nonetheless, high fidelity simulations are difficult due to the ocean's complex physics and broad range of spatial and temporal scales. These range from the smallest dissipative scales with typical lengths of millimeters, to the largest energetic mesoscale eddies with characteristic wavelengths exceeding tens of kilometers. Whereas the accurate parameterization of all flow scales with the Reynolds-averaged Navier-Stokes equations (RANS) is nearly impossible, resolving all scales of turbulence through a direct numerical simulation (DNS) is beyond the capabilities of the most powerful supercomputers in the foreseeable future. Hence, efficient parameterizations are needed. In this work, we extend the bridging partially-averaged Navier-Stokes equations (PANS) model to ocean flows. This parameterization operates between RANS and DNS, and aims only to resolve the scales not amenable to modeling in order to increase the efficiency of ocean computations. We also propose a PANS scale-aware closure to model the unresolved scales. The initial validation tests of the new PANS model include two representative ocean channel flows. The results confirm the potential of the new parameterization to predict oceanographically relevant turbulence efficiently.

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OCEAN TURBULENCE PREDICTION

CHALLENGE

Ocean flows are inherently turbulent and exhibit complex physics and wide range of scales. These range from the smallest and energy dissipative scales with characteristic length-scales of 1 – 10 mm, to the largest and energy-containing mesoscales, which characteristic length-scales can exceed 10 – 100 km. Thus the global ocean circulation flows possess phenomena smaller than a USA dime and phenomena larger than small countries (Fig. 1).

Resolving all flow phenomena through a direct numerical simulation (DNS) of the Navier-Stokes equations would require a grid with a minimum number of cells equal to 10^{21} [4]. Thus, performing a DNS of the global ocean circulation is beyond the capabilities of the most powerful computers in the foreseeable future.

An efficient turbulence model is needed!!!

TURBULENCE PARAMETERIZATION

The alternative to DNS is to parameterize the unresolved flow processes. Large-eddy simulation (LES) and the Reynolds-Averaged Navier-Stokes equations (RANS) are the most popular models:

- **LES:** leads to high-fidelity computations by resolving most of the turbulent scales. High computational cost.
- **RANS:** reduces the cost of LES significantly by parameterizing the entire turbulent spectrum (e.g., GM, GLS). Accurate closures are difficult to develop.

The limitations of LES and RANS led to the development of hybrid and bridging models. These scale-aware turbulence models significantly increase the efficiency of turbulence predictions by only resolving the phenomena not amenable to modeling (Fig. 2). The remaining scales are modeled by an adequate parameterization.

We recently extended the bridging partially-averaged Navier-Stokes equations (PANS) model to oceanic flows.

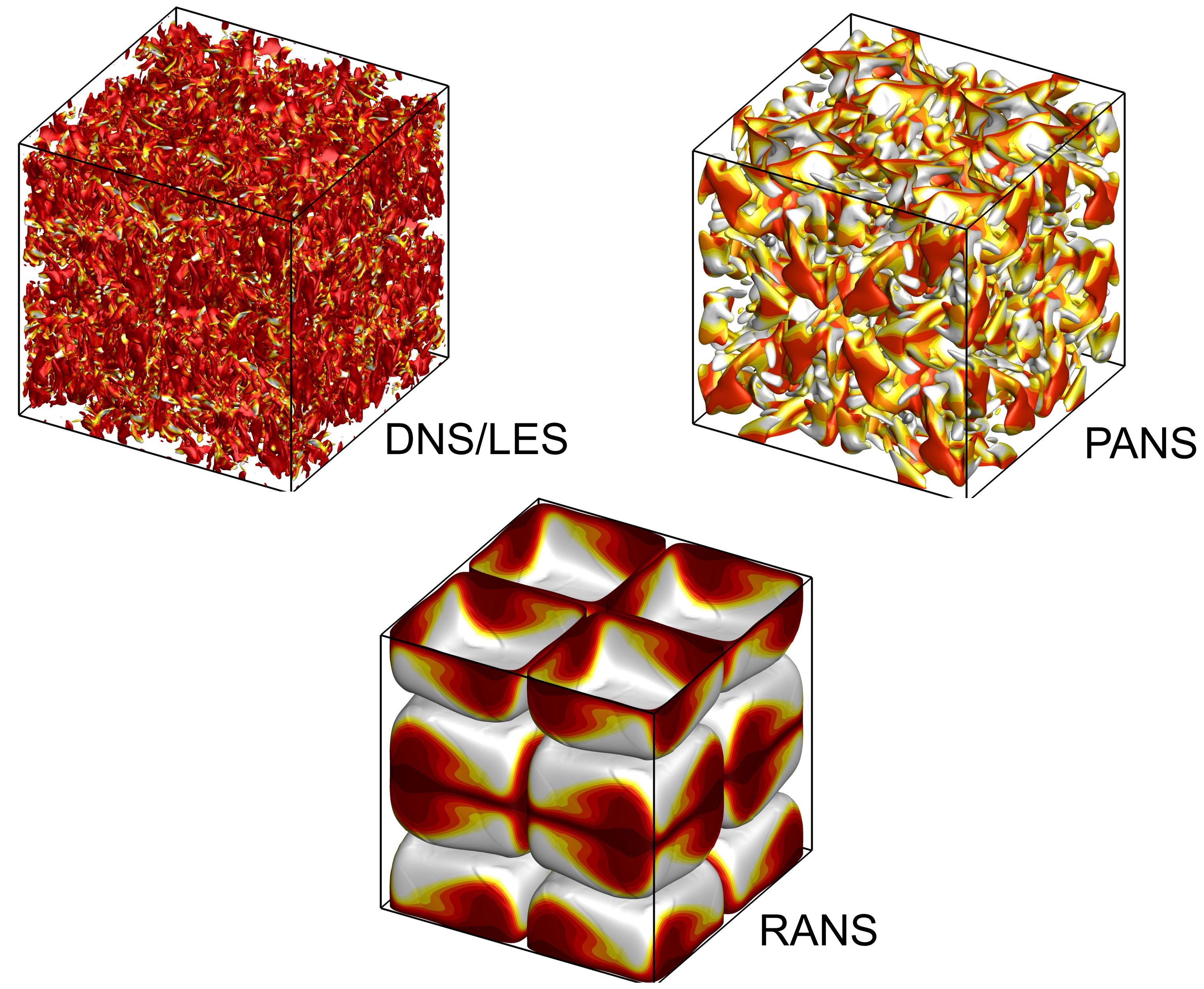

Figure 2. Resolved TGV flow scales by DNS, PANS, and RANS.

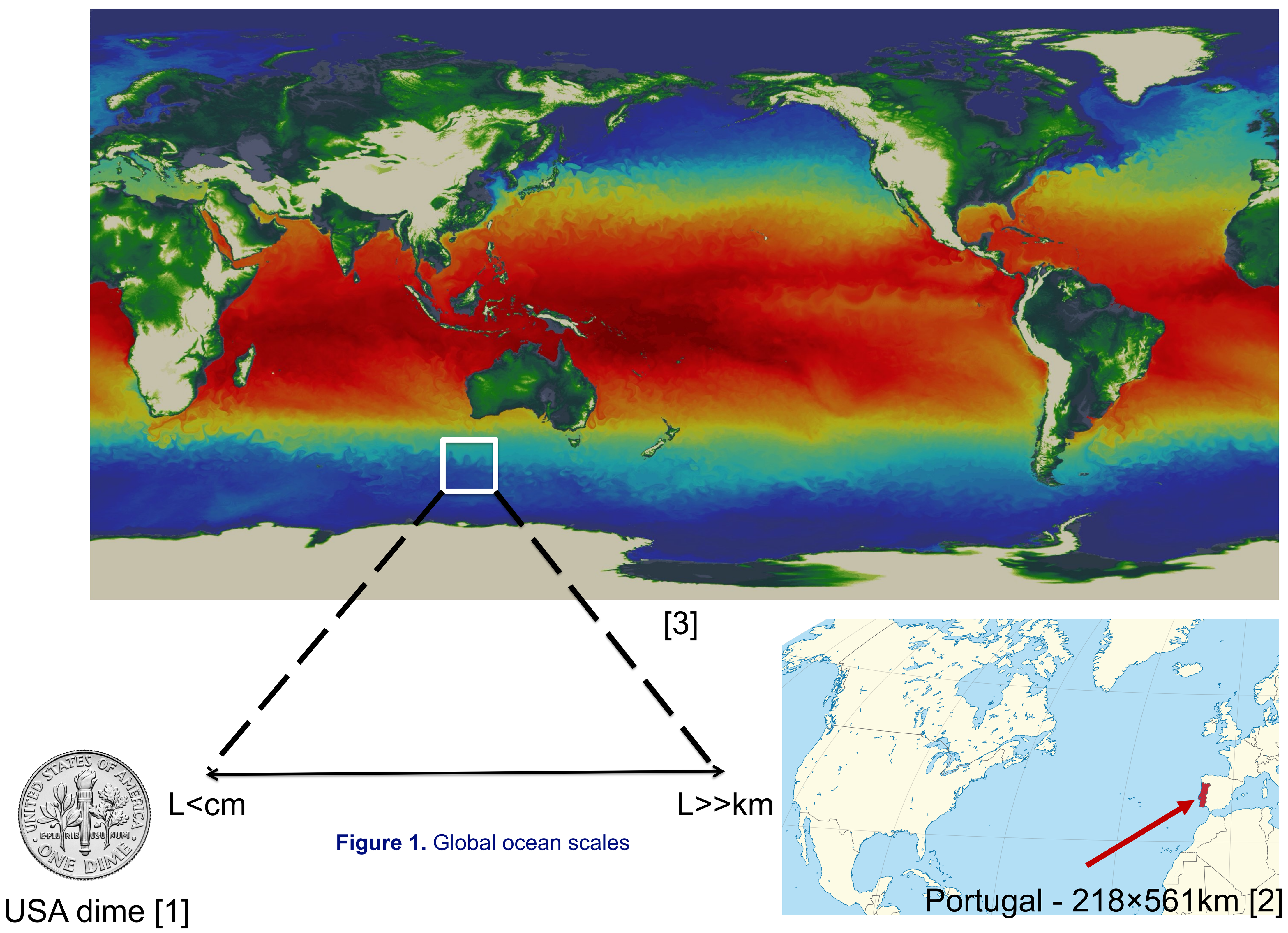

Figure 1. Global ocean scales

PARTIALLY-AVERAGED NAVIER-STOKES EQs

GOVERNING EQUATIONS & RESULTS

Bridging turbulence model, in which the range of resolved scales is set through a set of parameters, f_ϕ, defining the fraction of the turbulence dependent quantities ϕ being modeled:

$$\frac{\partial k_u}{\partial t} + \langle V_j \rangle \frac{\partial k_u}{\partial x_j} = \mathcal{P}_{s_u} + \mathcal{P}_{b_u} - \varepsilon_u + \frac{1}{\rho}\frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_u}{\sigma_k}\frac{f_\varepsilon}{f_k^2}\right)\frac{\partial k_u}{\partial x_j}\right]$$

PANS-GLS [4]

$$\frac{\partial \Psi_u}{\partial t} + \langle V_j \rangle \frac{\partial \Psi_u}{\partial x_j} = \frac{\Psi_u}{k_u}\left(c_1 P_{s_u} + c_3 P_{b_u}\right) - c_2^* \frac{\Psi_u \varepsilon_u}{k_u} + \frac{1}{\rho}\frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_u}{\sigma_k}\frac{f_\varepsilon}{f_k^2}\right)\frac{\partial \Psi_u}{\partial x_j}\right]$$

PANS can achieve LES accuracy at significantly lower cost (higher efficiency). For the TGV flow, the cost reduction exceeds 14 times. Validation space: Taylor-Green vortex (TGV), Rayleigh-Taylor (RT), channel flow (CF).

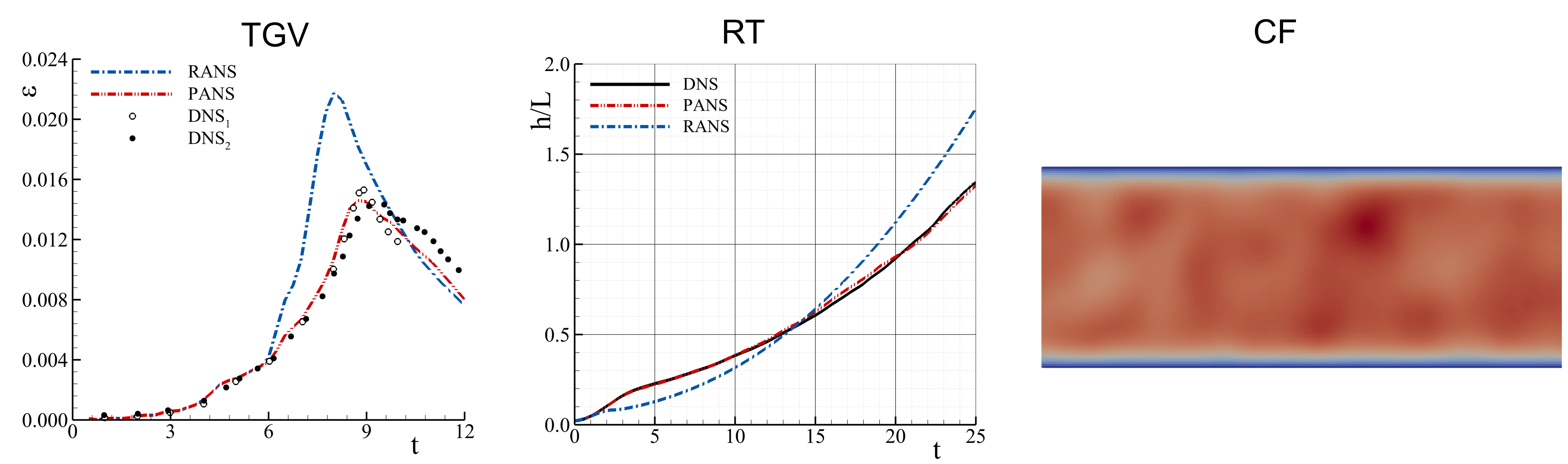

Figure 3. PANS simulations: TGV, RT, and CF.

[1] – https://en.wikipedia.org/wiki/Dime_(United_States_coin)#/media/File:2017-D_Roosevelt_dime_reverse_transparent.png
[2] – https://www.freeworldmaps.net/europe/portugal/location.html
[3] – J. Gafur and L. Van Roekel - Sea surface temperature from an eddy resolving E3SM forced ocean sea ice simulation.
[4] – F.S. Pereira, D.M. Israel, L. van Roekel, (A Novel Scale-Resolving Simulation Parameterization for Oceanic Turbulence (in progress)