

Dynamics of Electron-Scale Current Sheet Equilibria based on MMS Observations

David L. Newman¹, Giovanni Lapenta², and Martin V. Goldman¹

¹University of Colorado, Boulder CO

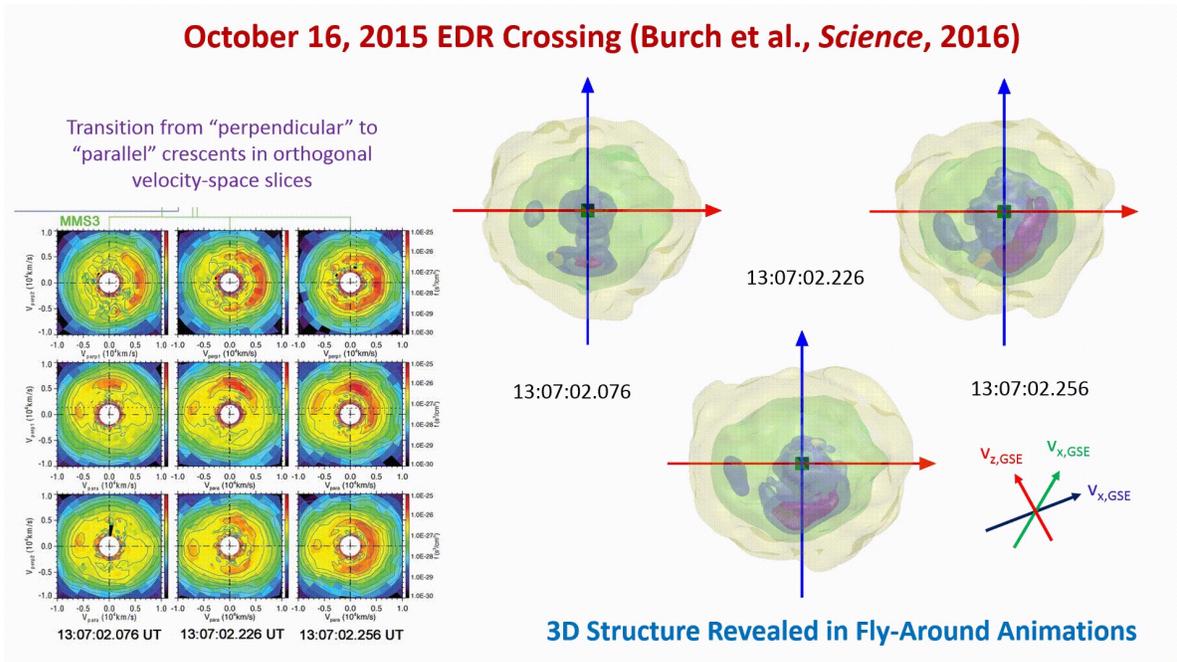
²KU Leuven, Belgium

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Abstract

The vicinity of the electron diffusion region (EDR) at the core of magnetic reconnection is frequently characterized by agyrotropic electron velocity distributions such as perpendicular velocity-space crescents [J. L. Burch et al., *Science*, DOI:10.1126/science.aaf2939 (2016)]. Although the evolving EDR is not itself an equilibrium state, its evolution may be slow on electron-cyclotron time scales. When this is the case, homogeneous equilibrium models are limited in their ability to model dynamical processes, such as instabilities and wave generation, in the presence of agyrotropic populations. In order to better study these processes, we initiate implicit PIC simulations with inhomogeneous kinetic equilibria built upon agyrotropic electron velocity distributions measured by the FPI spectrometers on MMS. The methodology involves the following elements: 1. Modeling the observed agyrotropic (e.g., crescent) and background plasma distributions 2. Numerically evaluating self-consistent inhomogeneous equilibria – including anisotropy 3. Initializing 1D, 2D, and 3D PIC simulations with the equilibria 4. Evaluating the simulation output for instabilities and the persistence of the crescents Particle tracing and other visualization tools will be employed to illustrate the underlying dynamics of particles and fields – and their interactions.

MOTIVATION AND OVERVIEW



The simultaneous observation by the four MMS satellites of agyrotropic electron distributions known as "crescents" were a highlight of the first detailed analysis of an electron diffusion region (EDR) crossing [Burch et al., 2016].

Challenge:

To incorporate observed crescent distributions into kinetic equilibria -- which are necessarily inhomogeneous due to the presence of agyrotropy. These equilibria can then be used to initialize kinetic simulations in order to study dynamical processes such as instability and wave generation.

Overview of Presentation Layout:

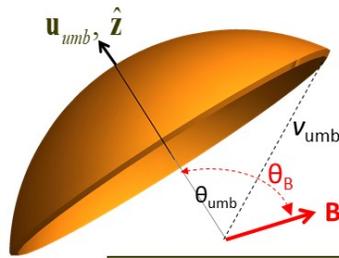
- Below: Methodology for approximating observed crescent distributions with model distribution functions that are readily incorporated into an inhomogeneous model
- Near-Right: Equilibrium solution based on the rightmost MMS3 distribution (above)
- Middle-Right: Kinetic simulations initialized with a simpler, but asymmetric, crescent-based equilibrium showing particle dynamics in 1D and subsequent evolution in different 2D planes
- Far-Right (upper): Gallery of high-resolution videos with frame control
- Far-Right (lower): Discussion and Future Goals

METHODOLOGY

Two distinct model distributions are considered, both of which are easily incorporated into a 1D inhomogeneous equilibrium solution. Observed distributions can be represented by a superposition of these model distributions, each of which is defined on a sphere of constant energy/velocity.

First model (waterbag crescent):

Crescents can be modeled as slices of 3-D umbrella velocity distribution



Umbrella in velocity-space spherical-coordinates

- Azimuthally symmetric about umbrella axis, z
- Delta function at radial speed, $v = |\mathbf{v}| = v_{umb}$
- Step function in polar angle, θ_{umb}
- No explicit dependence on B (not using field-aligned coordinates)

$$f_{umbrella}(v, \theta) = n_{umb} \frac{\delta(v - v_{umb})}{4\pi v^2} \frac{Step(\theta < \theta_{umb})}{1 - \cos(\theta_{umb})}$$

FLOW from electron number flux moment

$$\mathbf{u}_{umb}(v_{umb}, \theta_{umb}) = \hat{\mathbf{z}} v_{umb} \left[\frac{1 + \cos \theta_{umb}}{2} \right]$$

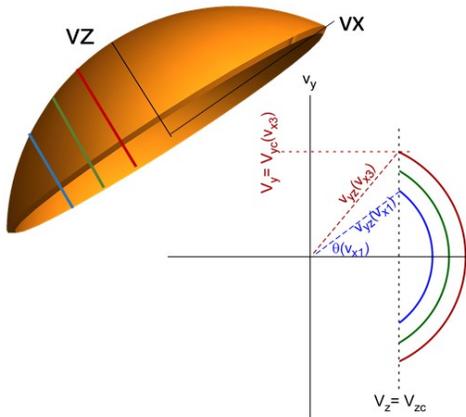
From Goldman presentation at MMS Workshop (Biarritz, Oct 2019)

Transport of Umbrella Perpendicular to B

Special case: $\mathbf{B} \parallel \mathbf{x}$ (i.e., $B_y = 0$)



v_x is a constant of motion



For "waterbag" distribution, Vlasov "operates" only on discontinuities in $f(\mathbf{v})$

$$v_{zc}(v_x) = v_{yz}(v_x) \cos \theta(v_x)$$

$$v_{yc}(v_x) = v_{yz}(v_x) \sin \theta(v_x)$$

$$\frac{d\theta}{dt} = \Omega \quad v_{yz} \equiv \text{const}$$

$$\frac{dv_{zc}}{dt} = -v_{yz} \Omega \sin \theta$$

$$\frac{dy}{dt} = v_{yz}$$



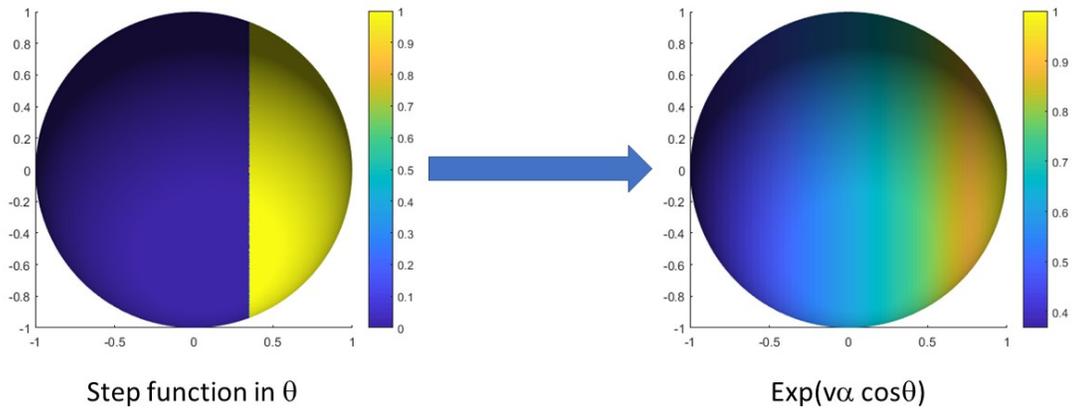
$$\frac{dv_{zc}}{dy} = -\Omega$$

Independent of v_x

Note: Addition of B_z (guide field) requires azimuthal symmetry about v_z

Second model (shaped crescent):

From Waterbag (Umbrella) to Shaped Crescent Model Distribution



Superpositions built on both model distributions can be readily incorporated when constructing an inhomogeneous equilibrium

The shaped crescent model is motivated by an analysis of drifting Maxwellians, which are fundamental to the well-known *Harris current-sheet* equilibrium

Harris Equilibrium: Prototype for Shaped Crescent Model (drifting Maxwellians)

$$f(\mathbf{v}) = N e^{-(v_x^2 + v_y^2 + [v_z - v_d]^2)/2v_{th}^2}$$

$$= N e^{-v_d^2/2v_{th}^2} e^{-v^2/2v_{th}^2} e^{v \cos(\theta) v_d/v_{th}^2}$$

Normalization

Shape

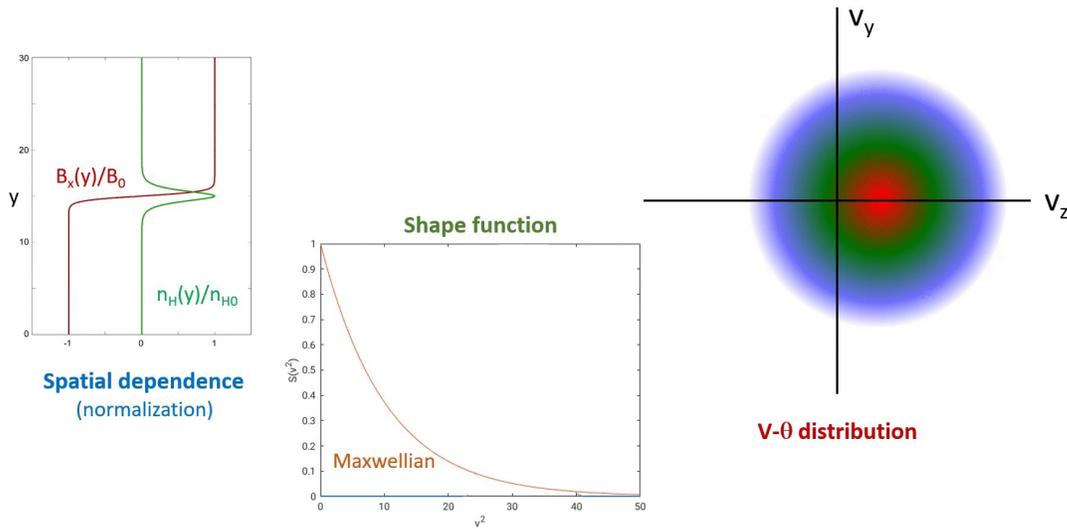
Essential Agyrotropy

(includes spatial dependence)

$$S(v^2)$$

$$\frac{v_d}{v_{th}^2} \equiv \alpha$$

From Drifting Maxwellian to Crescent



A simplified Vlasov analysis shows that a shaped-crescent distribution preserved its shape as it traverses a magnetic field perpendicular to the spatial gradient

Vlasov Equilibrium (1D-2V)

Time-independent Vlasov equation (ma=Lorentz force)

$$\mathbf{v} \cdot \nabla f = -\mathbf{a} \cdot \nabla_v f$$

Reduced dimensionality

$$f(\mathbf{x}, \mathbf{v}, t) \equiv f(y, v_y, v_z)$$

Polar coordinates (perpendicular to \mathbf{B})

$$v_y \equiv v \sin \theta; \quad v_z \equiv v \cos \theta; \quad \mathbf{B} \equiv B(y) \hat{x} \equiv \partial_y A_z(y) \hat{x}$$

Vlasov redux

$$v \sin \theta \cdot \partial_y f(y, v, \theta) = -\Omega_c(y) \cdot \partial_\theta f(y, v, \theta)$$

Solution:

$$\text{Ansatz : } f(y, v, \theta) = n(y) s(v) e^{v \alpha(v) \cos \theta}$$

$$n(y) = n(0) e^{q \alpha(v) A_z(y) / mc}; \quad A_z(0) = 0$$

$$s(v) \text{ and } \alpha(v) \text{ arbitrary}$$

Integration perpendicular to \mathbf{B} leads to a 1D spatially inhomogeneous equilibrium. A variant of this analysis can be employed for umbrella (waterbag) model distributions.

Procedure for Finding Kinetic Equilibrium (Vlasov + Maxwell)

Examples

Numerically integrate

$$\nabla \times \mathbf{A} = \mathbf{B}; \quad \nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J}$$

with

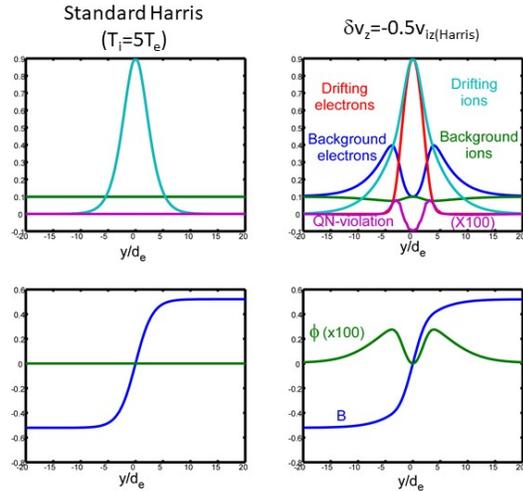
$$N_s(y) \sim e^{-q_s \alpha_s A_z(y)/m_s}$$

Electrostatic potential $\phi(y)$ can play key role

Where ϕ is adjusted to satisfy quasineutrality

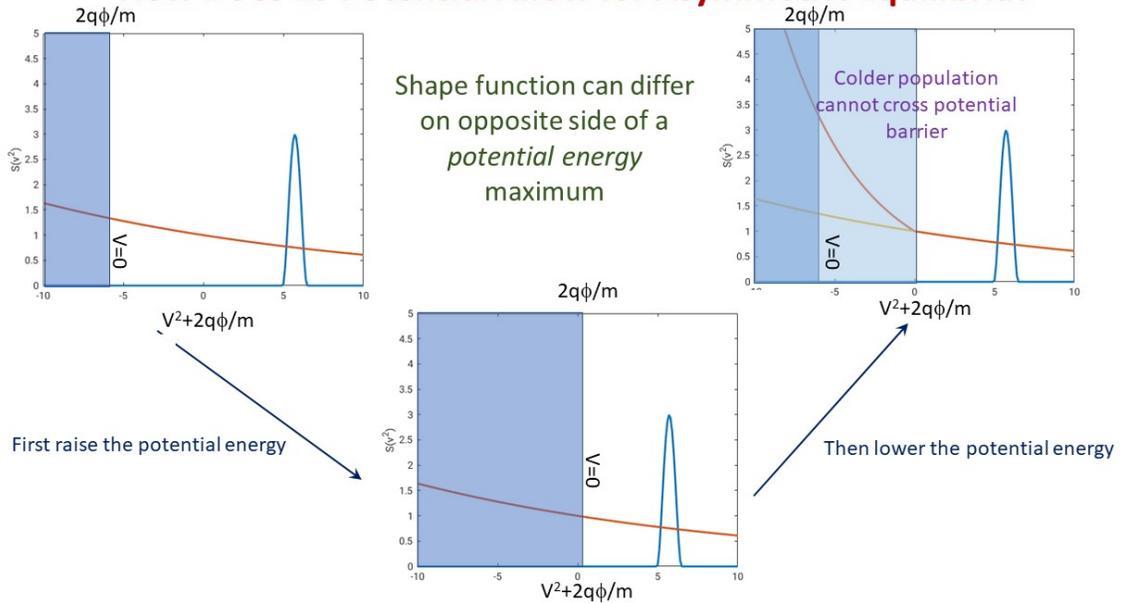
$$S(v^2, \phi) \rightarrow S(v^2 + 2q_s \phi/m_s)$$

[Boltzmann scaling for Maxwellians]



This method is not restricted to symmetric equilibria provided there is an electrostatic potential that can provide a barrier between different populations of a given species.

How Does ES Potential Allow for Asymmetric Equilibria?

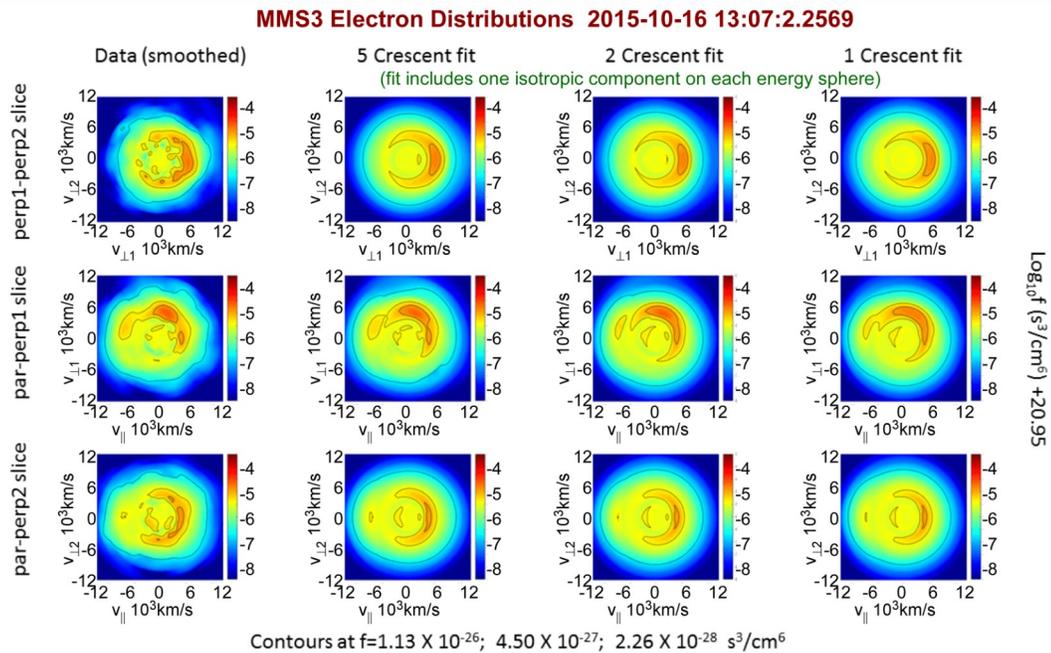


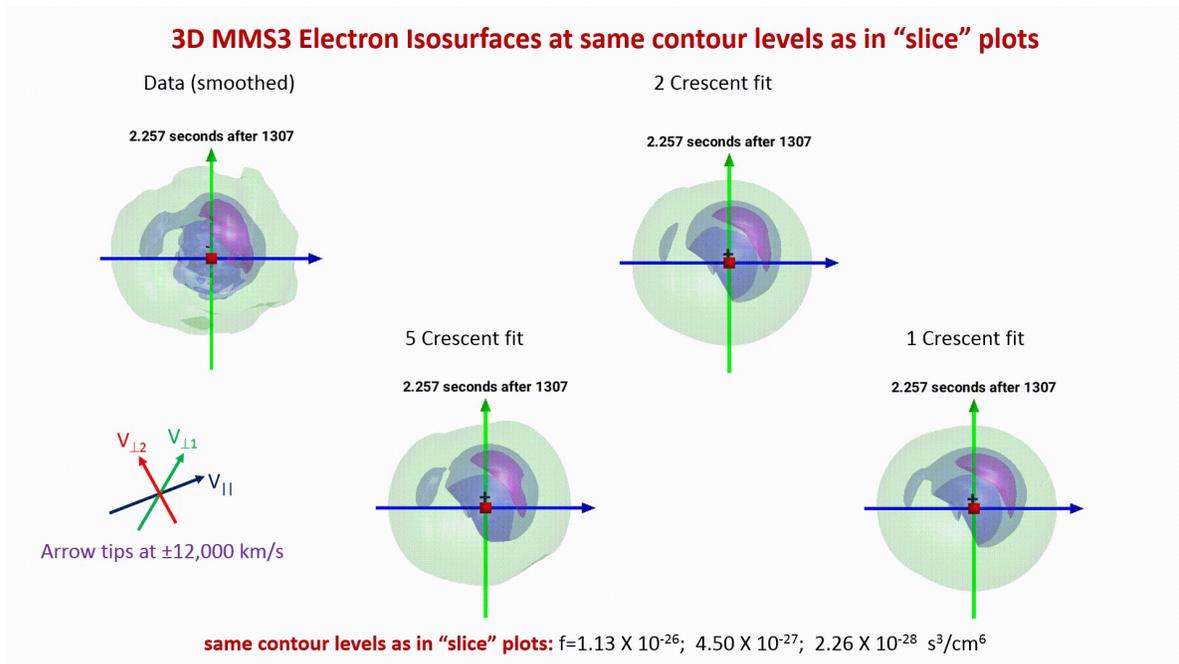
CONSTRUCTION OF EQUILIBRIUM BASED ON FIT TO OBSERVED ELECTRON CRESCENT DISTRIBUTION

Fitting Measured MMS Distribution with Shaped Crescent Model

- Fast Plasma Investigation (FPI) electron spectrometer (DES) generates “skymap” distribution function at 16 x 32 angles at 32 logarithmically spaced energies every 30ms
- After initial spherical harmonic smoothing, distribution on each energy “sphere” is subject to a nonlinear least-square fit consisting of one isotropic and prespecified number of shaped crescent components
- Each shaped crescent is characterized by a magnitude, shape parameter α , and orientation angle β in the $v_{||} - v_{\perp 1}$ plane
- Distribution is linearly interpolated between energy values

MMS3 distribution at 13:07:02.257 is a good candidate for approximating with a superposition of shaped crescent model distributions because of approximate symmetry in $v_{\perp 2}$ and approximate azimuthal symmetry about center of dominant crescent-like feature





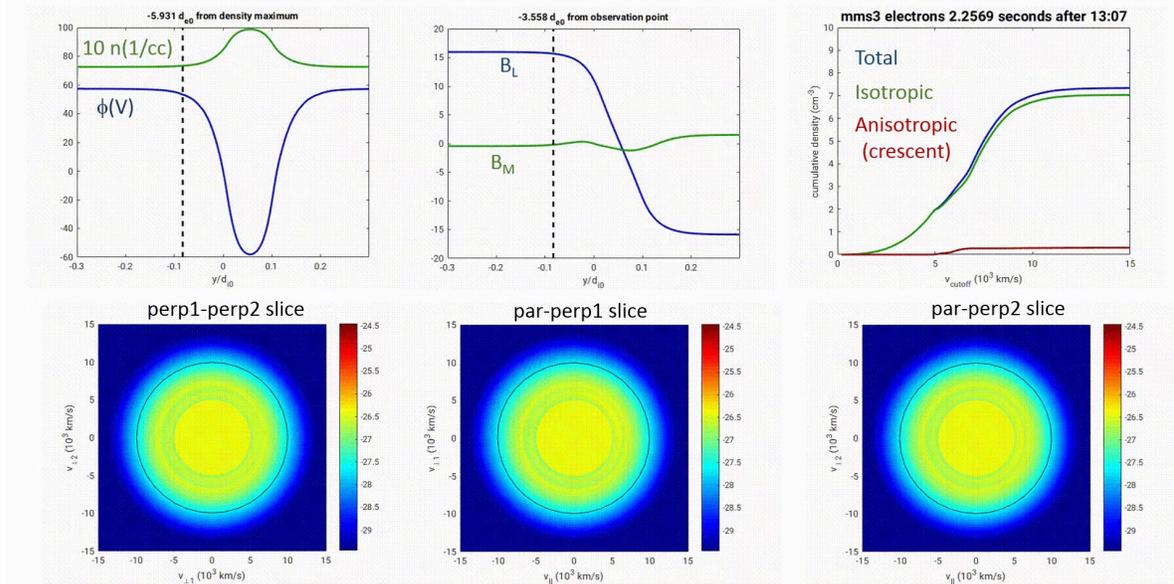
Improvement to the quality of the fit for >1 model crescent component at each energy is marginal.

Equilibrium Based on Observed MMS3 Distribution

Additional assumptions of model

- Fit to electron distribution consists of isotropic component plus *two* shaped crescent components
- Direction of crescents restricted to within ± 60 degrees of $v_{\perp 1}$
- Ions are modeled as a non-drifting Maxwellian distributions with temperature $T_i=375$ eV and Boltzmann dependence on ϕ
- For $\phi < 0$, low-energy electron distribution is modeled as a flat-top (no cold population)

Scan Across Equilibrium Based on Measured MMS3 Electron Distribution



Note: Apparent discontinuity in orientation of v_{\parallel} - $v_{\perp 1}$ distribution results from fact that $v_{\perp 1}$ component of flow vector is always positive

EVOLUTION OF PIC SIMULATION INITIALIZED WITH ASYMMETRIC CRESCENT-BASED EQUILIBRIUM

Parameters of Asymmetric Crescent-Based Current Sheet Used to Initialize Simulations

(Illustrative example – not based on specific observation)

➤ Crescent population:

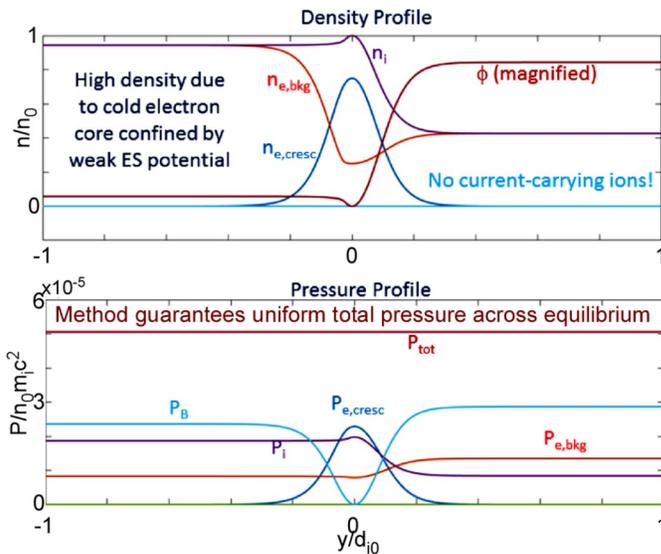
- Shape function $S(v)$ is quartic in kinetic energy localized over range $47.25 \times 10^3 \text{ km/s} < v < 54.00 \times 10^3 \text{ km/s}$
- Gyrotropy parameter $\alpha(v)$ is independent of v with value $\alpha = 5.35 \times 10^{-5} \text{ s/km}$ so that at maximum $S(v) f_{\text{cresc}} \sim e^{-2.7 \cos(\theta)}$
- Electron crescent balances 75% of ion density at center of current sheet ($y=0$), with remaining 25% consisting of background electrons

➤ Background populations (non-drifting)

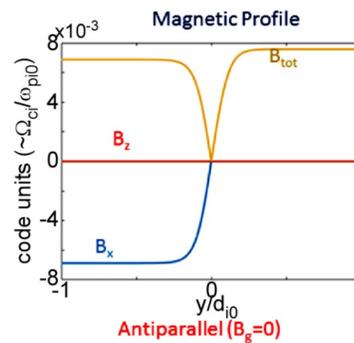
- Ions: 2.5 keV Maxwellian
- Electrons: 2.0 keV Maxwellian for $y > 0$ with a cold 7.8 eV core population for $y < 0$ (see discussion in Methodology section)

Note: $B=0$ at $y=0$ and determined elsewhere by integrating Ampere's Law without asymptotic boundary condition

Asymmetric Equilibrium Profile

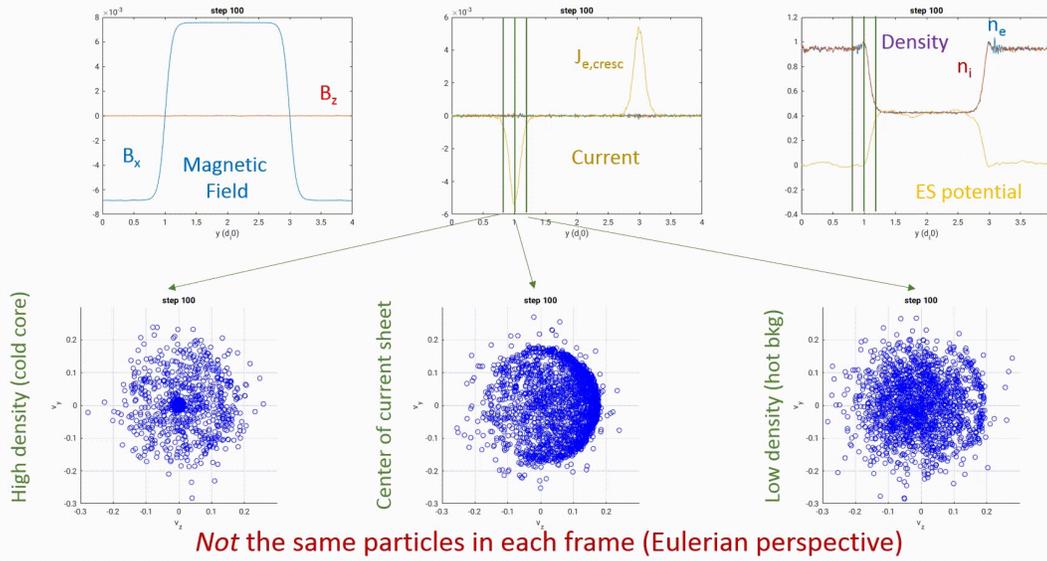


Simulations ($m_i/m_e=256$) are initialized with double (mirror symmetric) CS configuration allowing for periodic boundary conditions in y

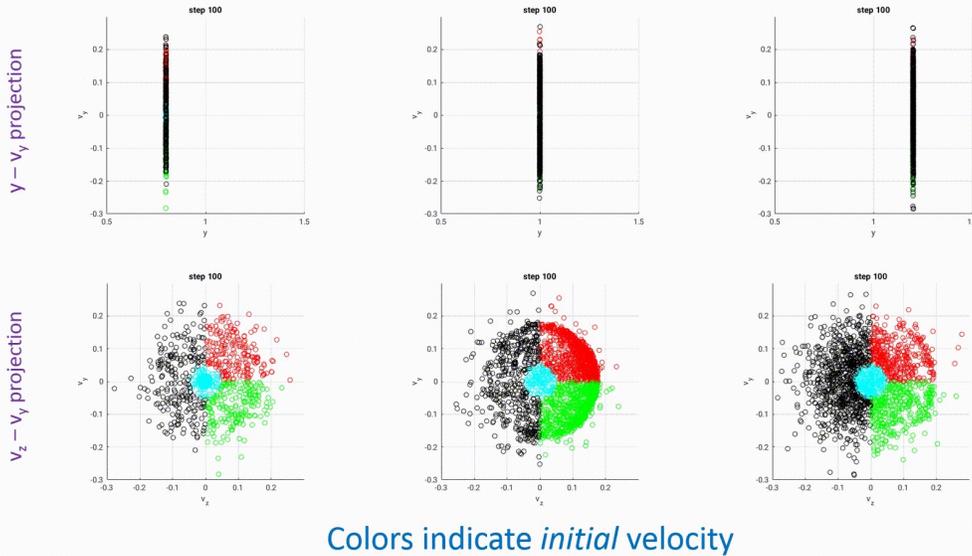


1D Simulations show that Initial distribution is an equilibrium state and illustrate difference in the dynamics of different subsets of electrons (e.g., localized vs meandering)

1D Current Sheet Evolution



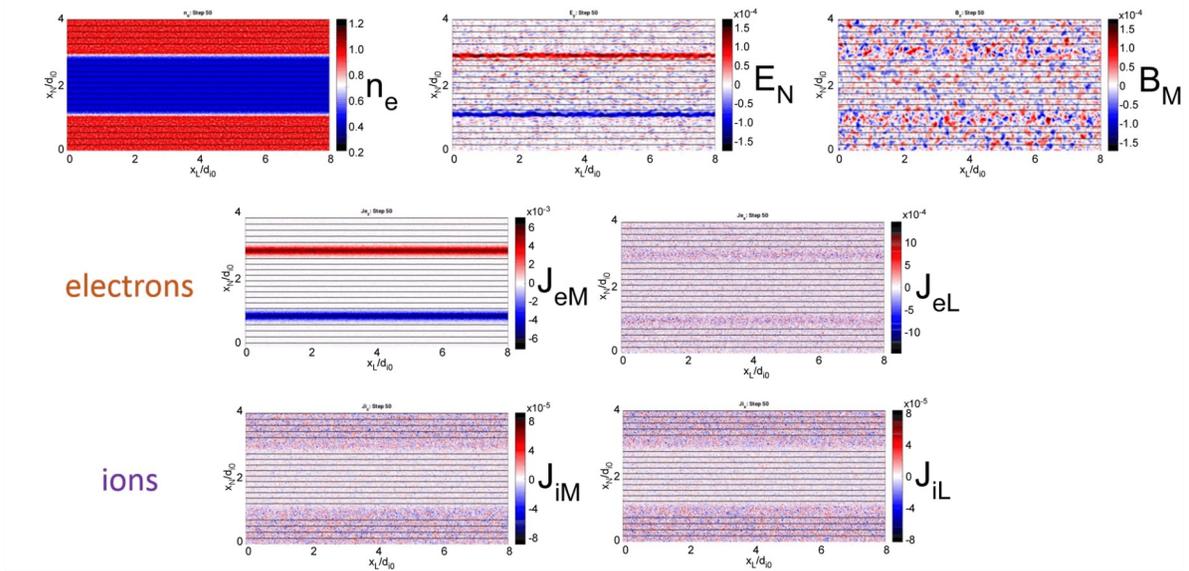
Tracking Fixed Sets of Electrons (Lagrangian Perspective)



2D simulations show that while the initial state is an equilibrium, it is nevertheless an unstable equilibrium subject to instabilities that depend on the plane in which the simulation is carried out.

2D Evolution in L-N Plane

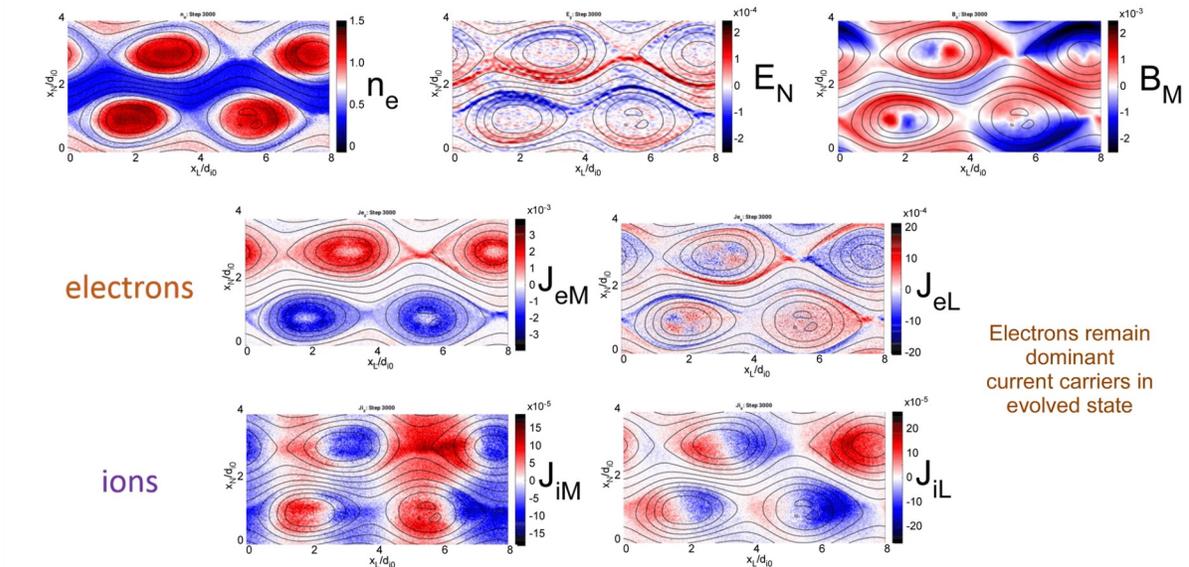
Initial State



Intermediate stages of evolution can be followed in the [Video Gallery](#).

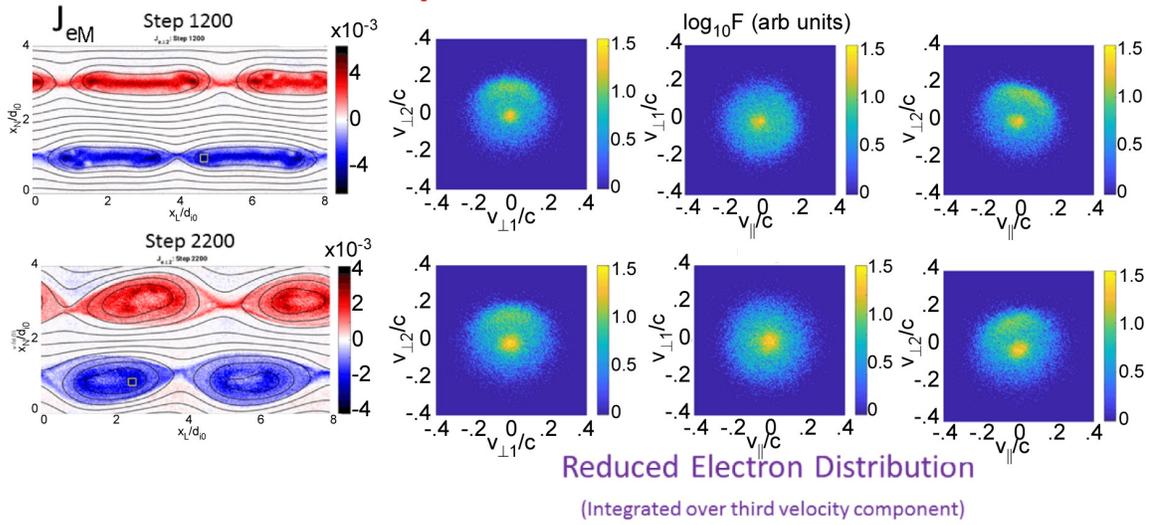
2D Evolution in L-N Plane

Late Stage



Electrons remain dominant current carriers in evolved state

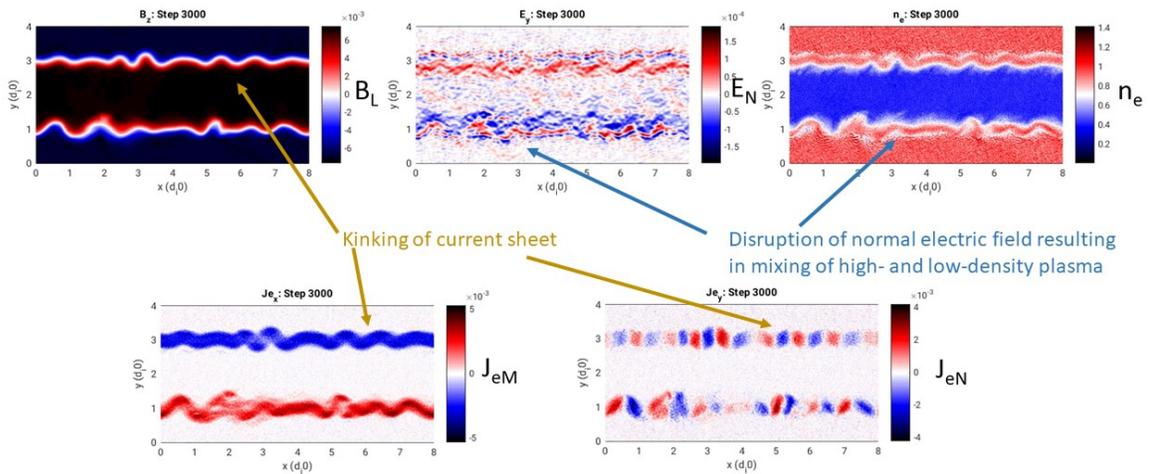
Crescent Distributions Persist in Regions of Strong Perpendicular Current



Rotating the initial distribution about the v_N axis is equivalent to rotating the simulation plane. Rotations through 90 and 45 degrees result in instabilities that differ from one another and from the simulations without rotation

2D Evolution – Distribution Rotated by 90 Deg

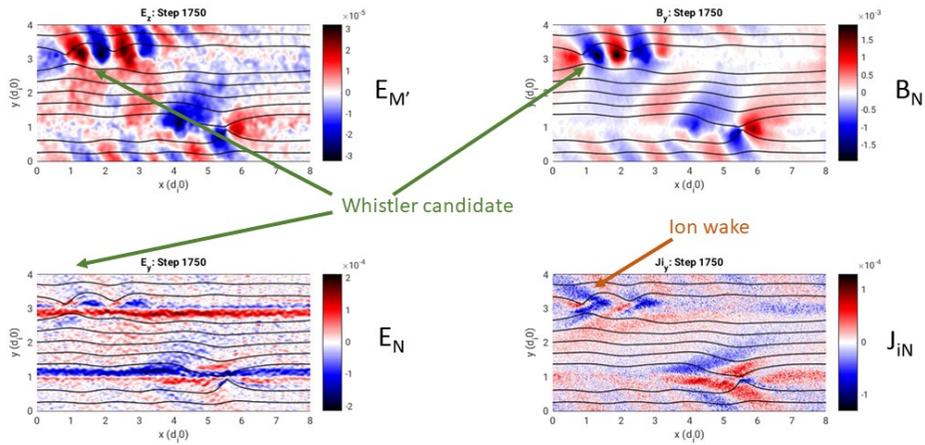
(Follow dynamical evolution in *Video Gallery*)



$X \rightarrow M; y \rightarrow N; z \rightarrow -L$

2D Evolution – Distribution Rotated by 45 Deg

(Follow dynamical evolution in *Video Gallery*)



$$X \rightarrow L' = (L+M)/2^{1/2}; \quad y \rightarrow N; \quad z \rightarrow M' = (M-L)/2^{1/2}$$

VIDEO GALLERY

Links to high resolution and controllable versions of all animations from this presentation are grouped here for convenient viewing.

[VIDEO] <https://www.youtube.com/embed/LTIiL9paXbc?feature=oembed&fs=1&modestbranding=1&rel=0&showinfo=0>

[VIDEO] <https://www.youtube.com/embed/D41xMNYICG8?feature=oembed&fs=1&modestbranding=1&rel=0&showinfo=0>

[VIDEO] <https://www.youtube.com/embed/Vgeq4T4Z7cY?feature=oembed&fs=1&modestbranding=1&rel=0&showinfo=0>

[VIDEO] <https://www.youtube.com/embed/RVFWHQs8dMA?feature=oembed&fs=1&modestbranding=1&rel=0&showinfo=0>

[VIDEO] <https://www.youtube.com/embed/NaozRidbUCI?feature=oembed&fs=1&modestbranding=1&rel=0&showinfo=0>

[VIDEO] <https://www.youtube.com/embed/IJnZayCYkVg?feature=oembed&fs=1&modestbranding=1&rel=0&showinfo=0>

[VIDEO] <https://www.youtube.com/embed/G6F-uwMOXxU?feature=oembed&fs=1&modestbranding=1&rel=0&showinfo=0>

[VIDEO] <https://www.youtube.com/embed/eB0qiCsfZyU?feature=oembed&fs=1&modestbranding=1&rel=0&showinfo=0>

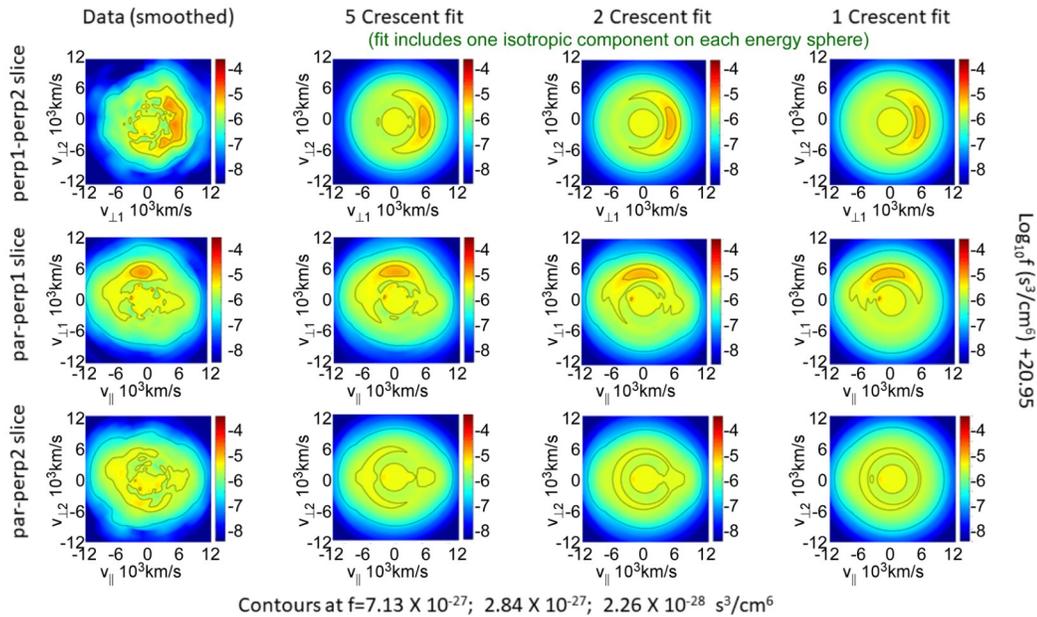
[VIDEO] <https://www.youtube.com/embed/s2W8SUIlgWE?feature=oembed&fs=1&modestbranding=1&rel=0&showinfo=0>

DISCUSSION AND FUTURE GOALS

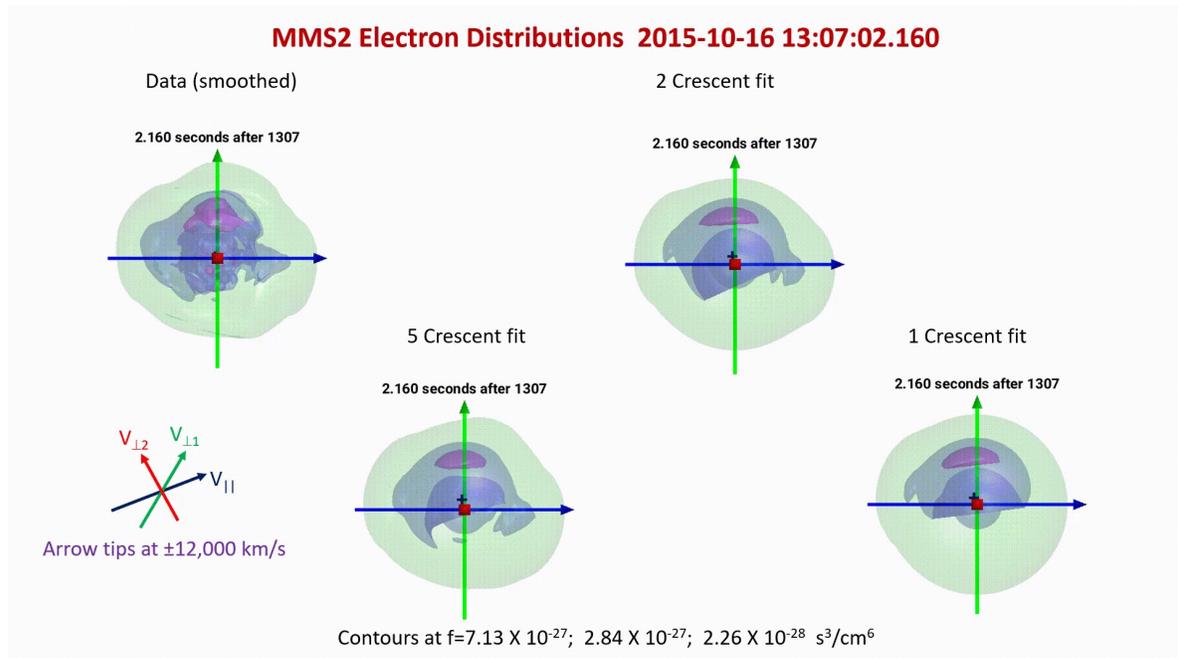
Not all observed crescent distributions are ideal candidates for the proposed fitting method.

The following distribution observed by MMS2 closely resembles the early MMS3 distribution at time 13:07:02.026 (see Motivation and Overview) in that the dominant crescent does not exhibit approximate azimuthal symmetry about its center.

MMS2 Electron Distributions 2015-10-16 13:07:02.160



MMS2 Electron Distributions 2015-10-16 13:07:02.160



A simpler model in which the magnetic field cannot rotate (with zero guide field) can accommodate such crescents, but has not been implemented yet.

Association between crescent-based equilibria and current sheets is *not* coincidental

If the crescent is the primary carrier of perpendicular current, the interaction of that current with the magnetic field (via Vlasov and Ampere) will lead to a reversal of the magnetic field in a region where the current is maximal.

The present study therefore shares aspects with previous investigations of generalized kinetic current-sheet equilibria, which often exhibit crescent-like features [see selected references].

Goals for Ongoing and Future Study

- Incorporate observation-based equilibria into PIC simulations
- Include more complex potential profiles with both maxima and minima to allow for multiple populations of both ions and electrons
- Run larger simulations in 2D as well as 3D
- Employ virtual satellite diagnostics in the simulations to compare with MMS observations

ABSTRACT

The vicinity of the electron diffusion region (EDR) at the core of magnetic reconnection is frequently characterized by agyrotropic electron velocity distributions such as perpendicular velocity-space crescents [J. L. Burch et al., Science, DOI:10.1126/science.aaf2939 (2016)]. Although the evolving EDR is not itself an equilibrium state, its evolution may be slow on electron-cyclotron time scales. When this is the case, homogeneous equilibrium models are limited in their ability to model dynamical processes, such as instabilities and wave generation, in the presence of agyrotropic populations. In order to better study these processes, we initiate implicit PIC simulations with inhomogeneous kinetic equilibria built upon agyrotropic electron velocity distributions measured by the FPI spectrometers on MMS.

The methodology involves the following elements:

- Modeling the observed agyrotropic (e.g., crescent) and background plasma distributions

- Numerically evaluating self-consistent inhomogeneous equilibria -- including anisotropy (as illustrated in the accompanying figure)
- Initializing 1D, 2D, and 3D PIC simulations with the equilibria
- Evaluating the simulation output for instabilities and the persistence of the crescents

Particle tracing and other visualization tools will be employed to illustrate the underlying dynamics of particles and fields -- and their interactions.

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