The Effect of Lorentz Stresses on the Solar Frequency Spectrum: The Forward Problem.

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November 23, 2022

Abstract

Departures from standard spherically symmetric solar models, in the form of perturbations such as global and local-scale flows and structural asphericities, result in the splitting of eigenfrequencies in the observed solar spectrum. Here we describe new theoretical developments that enable the computation of sensitivity kernels for frequency splittings (a coefficients) due to general Lorentz stresses in the Sun. We draw from theoretical ideas prevalent in normal-mode coupling theory in geophysical literature to build these kernels. We plot the Lorentz-stress kernels and estimate the a-coefficients arising from a combination of deep-toroidal and surface-dipolar fields (although we note that this could equally well be substituted by another choice of Lorentz stresses). These results pave the way to formally pose an inverse problem, and infer magnetic fields from the measured even a-coefficients.

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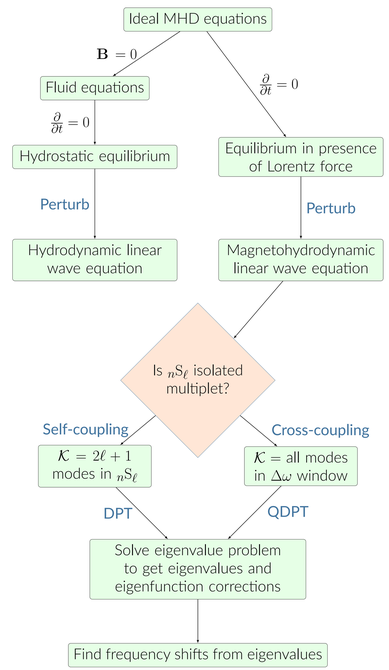
Abstract no: SH11D-3375*

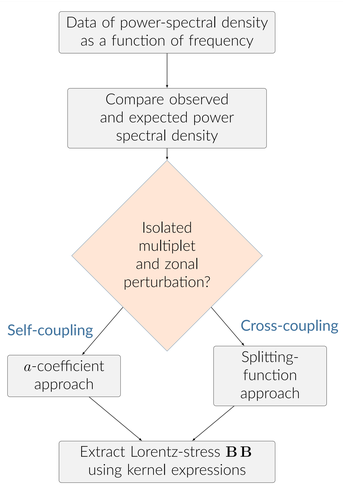
Abstract

Departures from standard spherically symmetric solar models, in the form of perturbations such as global and local-scale flows and structural asphericities, result in the splitting of eigenfrequencies in the observed spectrum of solar oscillations. We find the Lorentz-stress sensitivity kernel for a general magnetic field, and therefore, also propose the sensitivity kernels for frequency splittings (a- coefficients) due to axisymmetric Lorentz stresses in the Sun. These results pave the way to formally pose an inverse problem, and infer solar (stellar) internal magnetic fields.

Forward Problem: At a glance



Inverse Problem: In a blink



Magnetohydrodynamic (MHD) equations

$$\frac{\partial \rho}{\partial t} = -\boldsymbol{\nabla}\cdot(\rho\mathbf{v}), \quad (1)$$

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\boldsymbol{\nabla}\mathbf{v}\right) = -\boldsymbol{\nabla}p + \frac{\mathbf{j}\times\mathbf{B}}{c} - \rho\boldsymbol{\nabla}\phi, \quad (2)$$

$$\frac{\partial p}{\partial t} = -\mathbf{v}\cdot\boldsymbol{\nabla}p - \gamma p\boldsymbol{\nabla}\cdot\mathbf{v}, \quad (3)$$

$$\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla}\times(\mathbf{v}\times\mathbf{B}). \quad (4)$$

Definitions

GSH basis : $\hat{e}_- = \frac{1}{\sqrt{2}}(\hat{e}_\theta - i\hat{e}_\phi) \quad \hat{e}_0 = \hat{e}_r \quad \hat{e}_+ = -\frac{1}{\sqrt{2}}(\hat{e}_\theta + i\hat{e}_\phi)$

Expanding the displacement vectors in the basis of normal modes:

$$\boldsymbol{\xi}(\boldsymbol{r},\omega) = \sum_k \boldsymbol{\xi}_k e^{i\omega_k t}, = \sum_{st}\sum_\mu \xi^\mu_{st}(r) Y^\mu_{st}(\theta,\phi)\hat{e}_\mu$$

Figure 1:Spherical co-ordinate geometry

No toroidal components in $\boldsymbol{\xi}$. Similarly,

$$\mathbf{B} = \sum_{st}\sum_\mu B^\mu_{st}(r) Y^\mu_{st}(\theta,\phi)\hat{e}_\mu,$$

$$\boldsymbol{\mathcal{H}} = \mathbf{B}\,\mathbf{B} = \sum_{st}\sum_{\mu\nu} h^{\mu\nu}_{st}(r) Y^{\mu+\nu}_{st}(\theta,\phi)\hat{e}_\mu\hat{e}_\nu, \quad \mu\leftrightarrow\nu.$$

Background solar model: SNRNMAIS

We adopt the method of perturbation theory. Therefore, we need a background model of the Sun which shall be perturbed to `fit' the real Sun. It is standard practise in helioseismology to start with the following simplified model, referred to as **SNRNMAIS**:

- **S**pherically symmetric,
- **N**on-**R**otating,
- **N**on-**M**agnetic,
- **A**diabatic,
- **I**sotropic,
- **S**tatic.

We use Model-S.[1]

Modeling a custom magnetic field $\mathbf{B}$

- Inner toroidal upto tachocline, $r \le 0.7R_\odot$.
- Outer dipolar beyond $r \ge 0.95R_\odot$.
- Intermediate mixed (toroidal + spheroidal) field.

$$B_{st}(r) = \begin{cases} -i\frac{\alpha(r)}{\gamma_1}\begin{pmatrix}1\\0\\-1\end{pmatrix} - \frac{\beta(r)}{\gamma_1}\begin{pmatrix}b - r\dot{b}\\-2b\\b - r\dot{b}\end{pmatrix}, & \text{for}(s,t) = (1,0)\\ 0, & \text{for}(s,t)\neq(1,0),\end{cases}$$

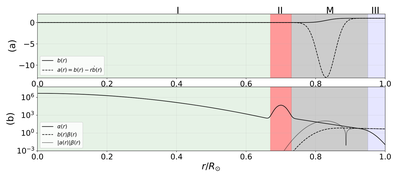
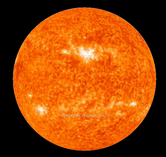


Figure 2:(a) Profiles for tunable parameters $a(r)$ and $b(r)$ chosen in this study. (b) Strength associated to each GSH component of B^μ_{10}.

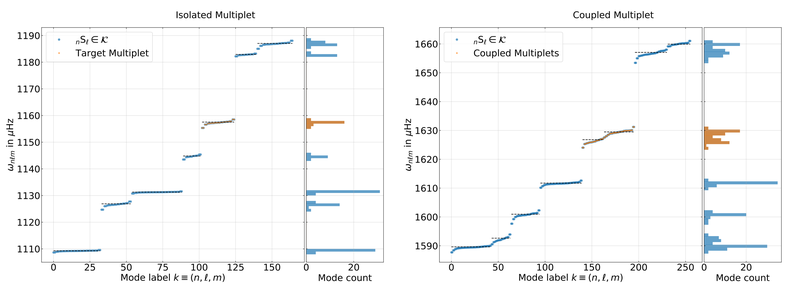
How to find **Sun's internal magnetic fields** by observing the solar **surface oscillation frequency**?

1. Ideal MHD,
2. Normal-mode helioseismology,
3. Quasi-degenerate perturbation theory.



Isolated multiplet vs. coupled multiplet

$$\sum_k\{\Lambda_{k'k} - (\omega^2_{\rm ref} - \omega^2_k)\delta_{k'k}\}c_k = \sum_k \delta(\omega^2)\delta_{k'k}c_k \quad \text{where,} \quad \Lambda_{k'k} = \sum_{st}\sum_{\mu\nu}\int_0^{R_\odot} dr\, r^2 \mathcal{B}^{\mu\nu}_{st}(r) h^{\mu\nu}_{st}(r). \quad (5)$$



Labelling multiplets: Isolated or coupled?

$$L^{QDPT}_2 = \sqrt{\sum_m(\delta\omega^Q_{n\ell m})^2} \quad \text{for cross-coupling,} \quad L^{DPT}_2 = \sqrt{\sum_m(\delta\omega^D_{n\ell m})^2} \quad \text{for self-coupling.} \quad (6)$$

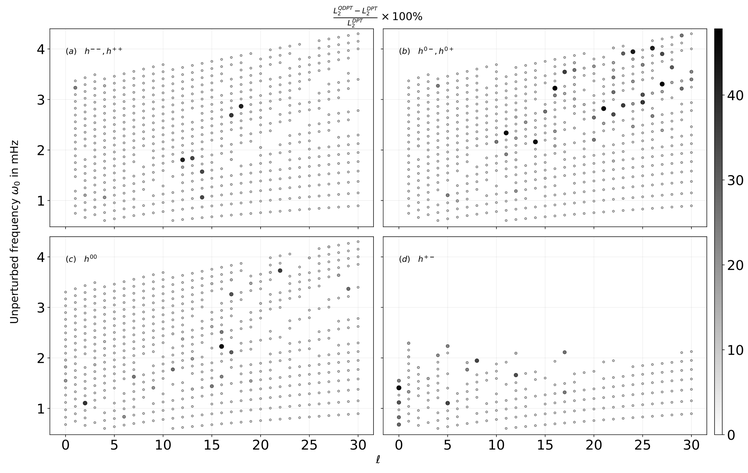


Figure 3:Darker and larger `o' represents multiplets where cross-coupling causes significant relative frequency offset with respect to self-coupling. We have used Lorentz-stress field with $s = \{0,1,2,3,4,5,6\}$ including all t for a certain s. The strength of GSH components $h^{\mu\nu}_{st} = h_0(r)$ where $h_0(r)$ is the `total' field strength as shown in Figure 4.

Inversion technique using a-coefficients

- Isolated-multiplet approximation,
- Axisymmetric perturbation $\implies t = 0$,
- Extensively used for inversion of differential rotation.[2]

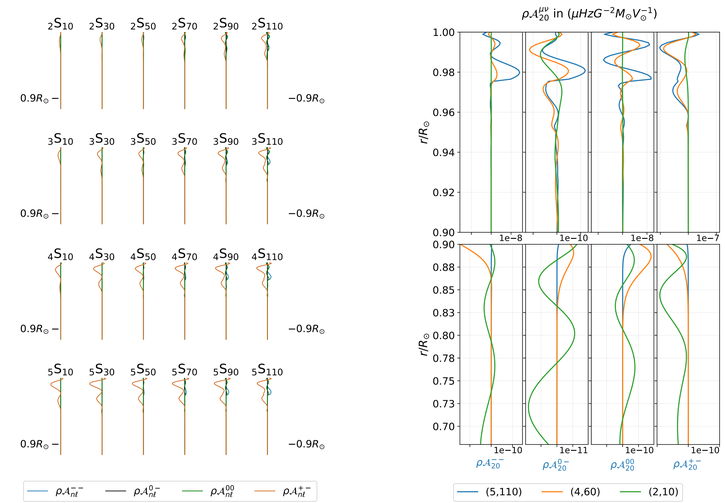
The inverse problem $\longrightarrow$

$$\delta\omega^m_{n\ell} = \sum_{j=0}^{j_{max}} a^{n\ell}_j P^{(\ell)}_j(m),$$

$$a^{n\ell}_s = \int_0^{R_\odot} dr\, r^2 \sum_{\mu,\nu}\mathcal{A}^{\mu\nu}_s h^{\mu\nu}_{s0},$$

$$\text{where,} \quad \mathcal{A}^{\mu\nu}_s(r) = \frac{\mathcal{K}^{\mu\nu}_s(r)}{2\omega_{n\ell}} \propto \frac{\mathcal{B}^{\mu\nu}_{s0}(r)}{2\omega_{n\ell}}.$$

Self-coupling kernels for a-coefficients



Varying depth sensitivity of modes

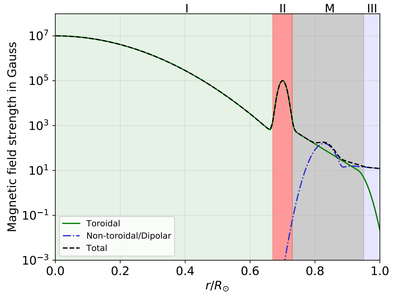


Figure 4:Magnetic field strength (Model M1 in Table 1) associated with three different regions with the following field configurations: (A) Purely toroidal for $r < 0.7R_\odot$, (B) Toroidal + spheroidal extending from $r = 0.7R_\odot \le r \le 0.95R_\odot$ and (C) Purely dipolar for $r > 0.95R_\odot$.

	M1	M2	M3	M4	M5	M6	M7	M8
B_I	Low	Low	Low	Low	High	High	High	High
B_{II}	Low	Low	High	High	Low	Low	High	High
B_{III}	Low	High	Low	High	Low	High	Low	High

Table 1:Strength ratio High:Low = 10:1. M1 = Sun-like.

Modes	${}_2S_{10}$		${}_5S_{110}$	
a-coefficients	a_0	a_2	a_0	a_2
M1	4.138	-0.501	2.399	0.628
M2	4.991	-0.167	239.986	62.863
M3	399.696	-62.360	2.399	0.628
M4	400.549	-62.026	239.986	62.863
M5	14.274	11.824	2.399	0.628
M6	15.127	12.158	239.986	62.863
M7	411.551	-50.349	2.399	0.628
M8	412.405	-50.015	239.986	62.863

Table 2:Mode-response in a-coefficients (nHz).

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$$\mathcal{B} = \frac{1}{2}\left\{\nabla\boldsymbol{\xi}_k\cdot(\nabla\boldsymbol{\xi}^*_{k'})^T + \nabla\boldsymbol{\xi}^*_{k'}\cdot(\nabla\boldsymbol{\xi}_k)^T\right\} + \frac{1}{2}\left\{\nabla\boldsymbol{\xi}^*_{k'}\cdot\nabla\boldsymbol{\xi}_k + \nabla\boldsymbol{\xi}_k\cdot\nabla\boldsymbol{\xi}^*_{k'}\right\} + \frac{1}{2}\left\{\boldsymbol{\xi}^*_{k'}\cdot\nabla\nabla\boldsymbol{\xi}_k + \boldsymbol{\xi}_k\cdot\nabla\nabla\boldsymbol{\xi}^*_{k'}\right\} - \frac{1}{2}\left\{\boldsymbol{\xi}_k\nabla\nabla\cdot\boldsymbol{\xi}^*_{k'} + \boldsymbol{\xi}^*_{k'}\nabla\nabla\cdot\boldsymbol{\xi}_k\right\} - \frac{3}{2}\left\{\nabla\boldsymbol{\xi}_k\nabla\cdot\boldsymbol{\xi}^*_{k'} + \nabla\boldsymbol{\xi}^*_{k'}\nabla\cdot\boldsymbol{\xi}_k\right\} + \boldsymbol{I}\nabla\cdot\boldsymbol{\xi}^*_{k'}\nabla\cdot\boldsymbol{\xi}_k$$