Using Atomic Clocks and Quantum Gradiometers Onboard Satellites for Determining the Earth's Gravity Field

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Abstract

Satellite missions like GRACE (now followed by GRACE-FO) and GOCE have remarkably advanced our knowledge on the global Earth's gravity field, by measuring the first and second derivatives of the gravitational potential. However, a more precise gravity field model with better spatial and temporal resolution is still highly required by various geoscience disciplines such as oceanography, solid Earth physics, geodesy, etc. New technologies based on quantum optics emerged and quickly developed in the past years. They will enable novel observation concepts and deliver gravimetric observations with an unprecedented accuracy level in future. For the first time, optical clocks provide the particular opportunity to directly observe gravity potential differences through measuring the relativistic redshift between clocks connected by dedicated links ("relativistic geodesy"). Moreover, cold atom interferometry and optical gradiometers have extensively been studied. They will potentially provide gravity gradient measurements with an accuracy of about one order of magnitude better than the electrostatic gradiometer that was used in GOCE. To figure out how these future gravimetric observations may benefit the modelling of the Earth's gravity field, we ran simulations using multi-source data, including gravity gradients, gravity accelerations and (satellite-based) clock measurements. Estimated instrument errors are mapped to the gravity field coefficients. Additionally, the individual contribution of each type of the new observations is evaluated, including its spectral behavior. Our results indicate that resulting gravity field solutions might be one order of magnitude more accurate than the current satellite-only models.

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Motivation

In the past decades, satellite missions like GRACE and GOCE have advanced our knowledge on the Earth's gravity field, by measuring the first- and second-order derivatives of the gravitational potential. However, a more precise gravity field model with a better spatio-temporal resolution is still highly demanded for geodetic and further geoscience applications. In recent years, new technologies based on quantum optics emerged and quickly developed, which will enable novel observation concepts and deliver gravimetric observations with an unprecedented accuracy in future. For the first time, atomic clocks provide a particular opportunity to directly observe gravity potential differences through measuring the relativistic redshift between clocks ("relativistic geodesy"). A quantum gradiometer, e.g., the Cold Atom Interferometry (CAI) gradiometer, is expected to deliver gravity gradients with an accuracy of about one order of magnitude higher than that of GOCE. The contribution of these quantum sensors to improve the Earth's gravity field are evaluated, where the instrumental errors are mapped to the gravity field coefficients through closed-loop simulations.

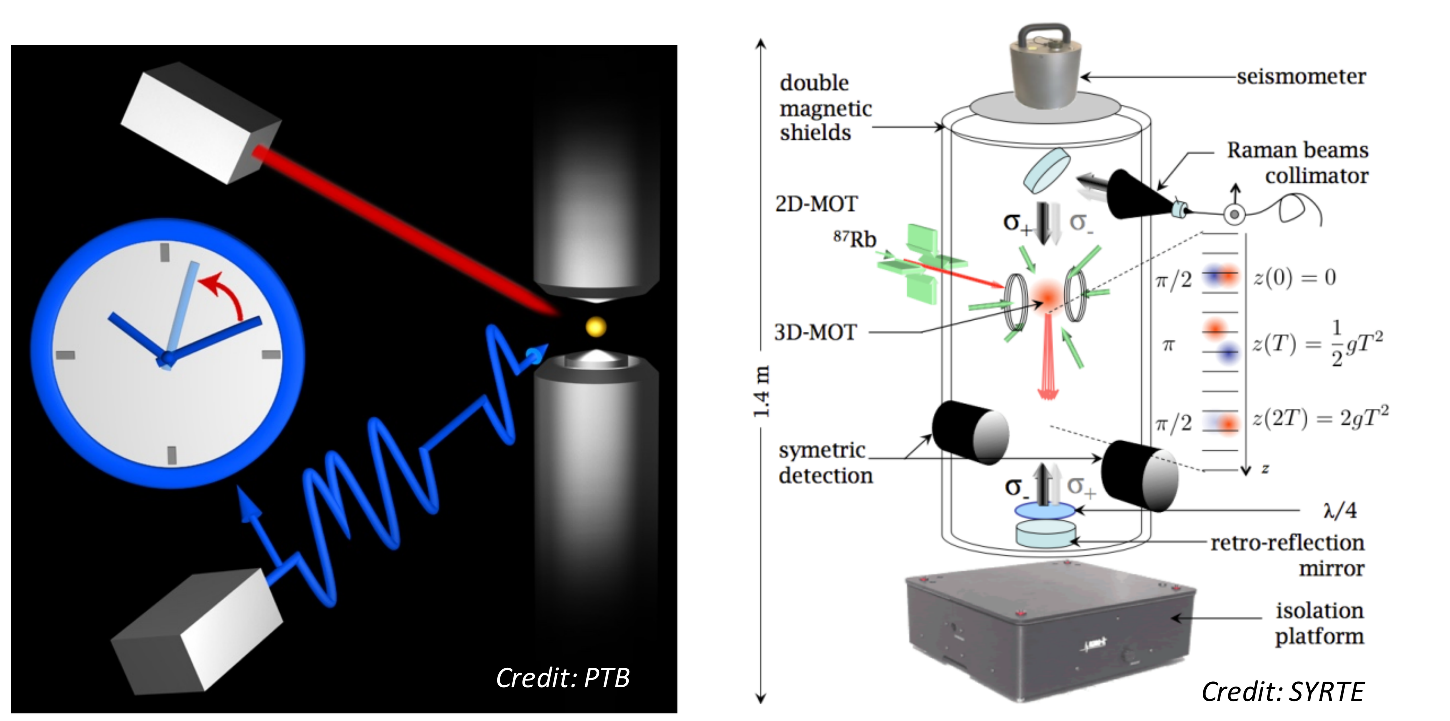

Fig. 1: The atomic clock e.g., a single-ion optical clock (left) and the Cold Atom Interferometry (CAI) gradiometer (right).

Retrieving the Earth's gravity field

The global gravity field is expressed as

$$T = \frac{GM}{R}\sum_{n=0}^{\infty}\left(\frac{R}{r}\right)^{n+1}\sum_{m=-n}^{n}\bar{K}_{nm}\,\bar{Y}_{nm}(\theta,\lambda)\,,$$

$$\bar{Y}_{nm}(\theta,\lambda) = \bar{P}_{nm}(cos\theta)e^{im\lambda}.$$

It can be retrieved by observing

- potential values (T);
- gravity accelerations ($T_i = \frac{\partial T}{\partial r_i}$);
- gravity gradients ($T_{ij} = \frac{\partial^2 T}{\partial r_i\, \partial r_j}$).

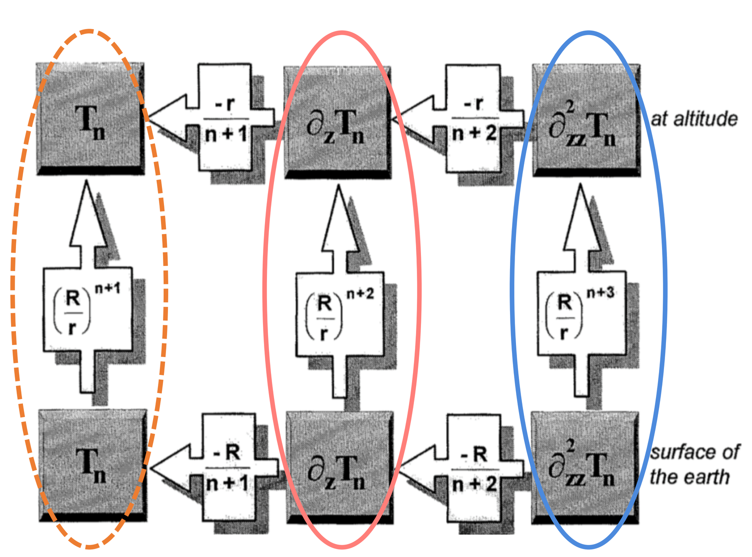

Fig. 2: Extended Meissl scheme. Detecting the Earth gravity field by observing the zero-, first- and second-order derivatives of gravity potential in space.

Closed-loop simulator

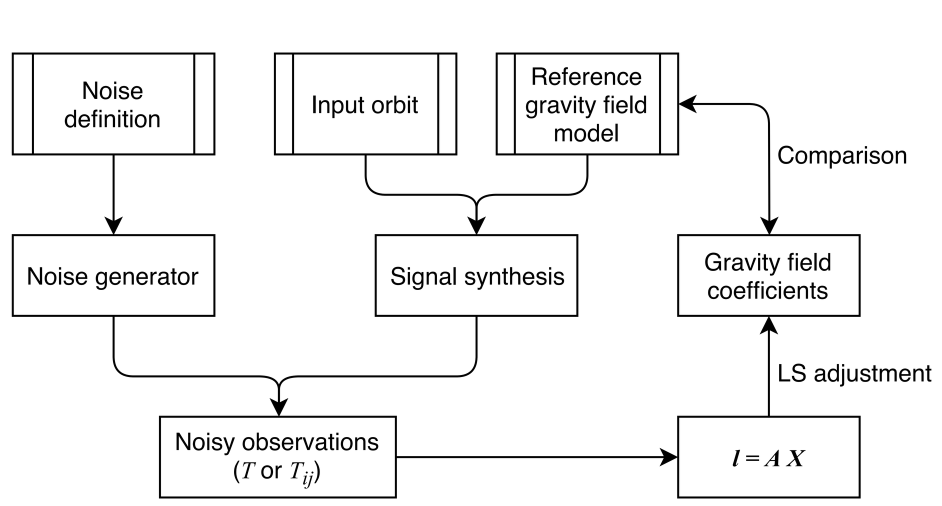

Fig. 3: Scheme of our closed-loop simulator for gravity field recovery from clock and CAI data. The observation signals are synthesized from a background model, e.g., EIGEN-6c4. The noise is generated based on the specifications of the sensor behavior. A rigorous Least-Squares (LS) adjustment is applied to retrieve the gravity field coefficients, which are compared to the input model for evaluation.

Atomic clocks

- **Basis:** Einstein's general theory of relativity;
- **Gravitational redshift:** $\frac{\Delta f_{21}}{f_1} = \frac{f_2 - f_1}{f_1} = \frac{W_2 - W_1}{c^2} + O\,(c^{-4})$;
- **Error propagation:** $\frac{\Delta f}{f}\ (1.0 \times 10^{-18}) \sim \Delta W\ (0.1\ \mathrm{m^2/s^2}) \sim \Delta h\ (1.0\ \mathrm{cm})$.

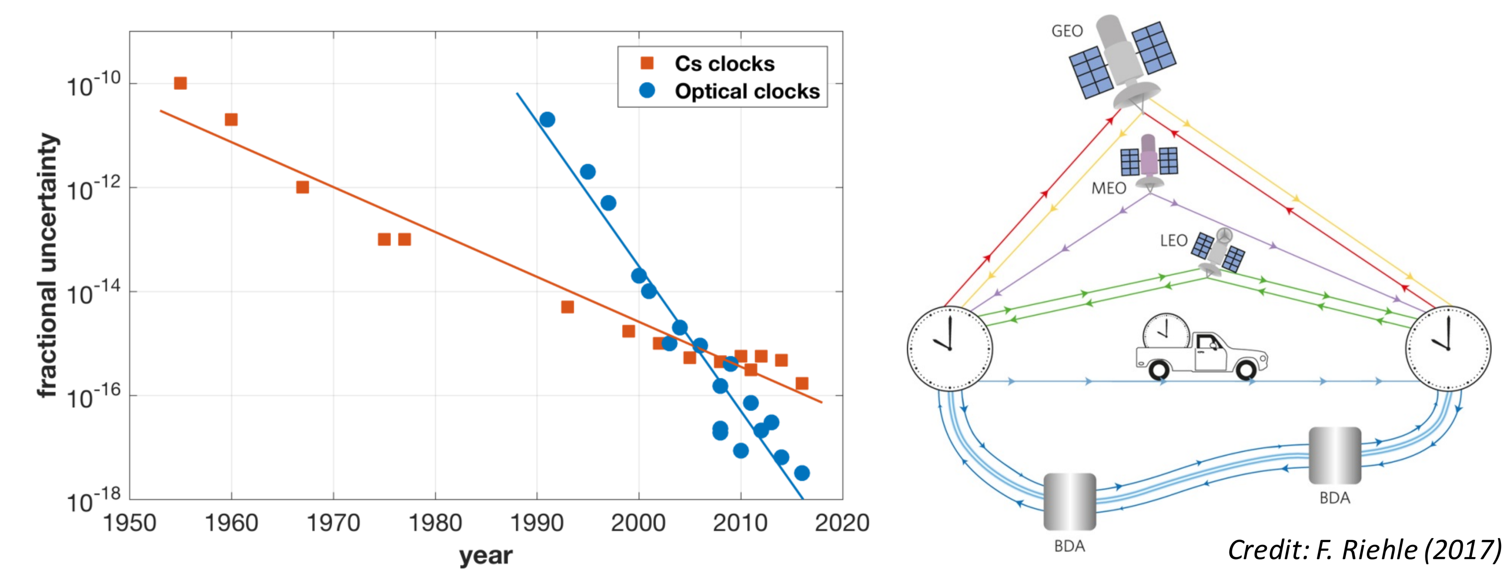

Fig. 4: Evolution of atomic clocks' performance (left) and various frequency link techniques (right). The frequency comparison between distant clocks is now approaching the accuracy level of 1.0×10^{-18}, which can be translated to the potential differences at the level of 0.1 m²/s².

Input for simulation

- Orbit: GOCE, 2 months (Nov. and Dec., 2009), 5s;
- Model: EIGEN-6c4, d/o 360;
- Noise: white, with different levels.

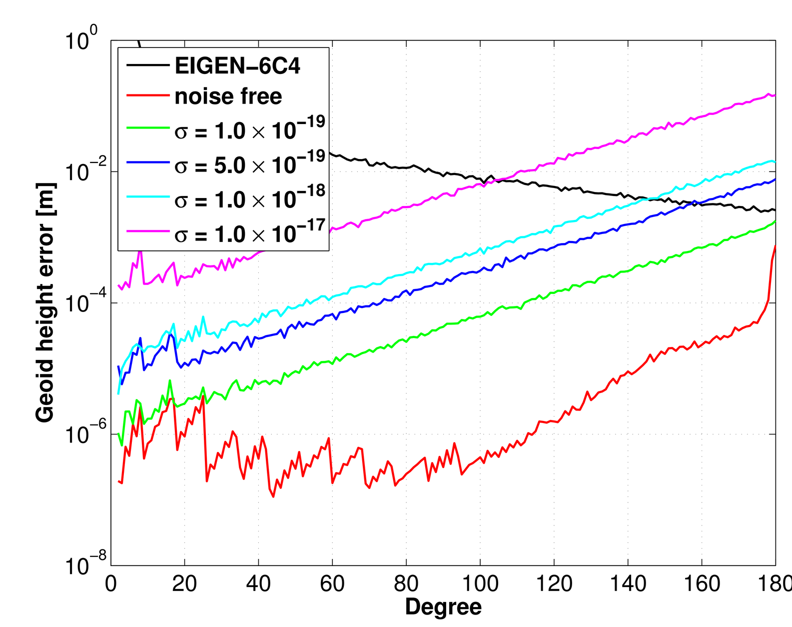

Fig. 5: Degree variances of gravity field coefficient differences w.r.t. EIGEN-6c4, in terms of geoid height. The gravity field models were recovered up to d/o 180. The zonal and near-zonal coefficients that are degraded by the polar gaps of the GOCE orbit have been excluded to compute these degree variances.

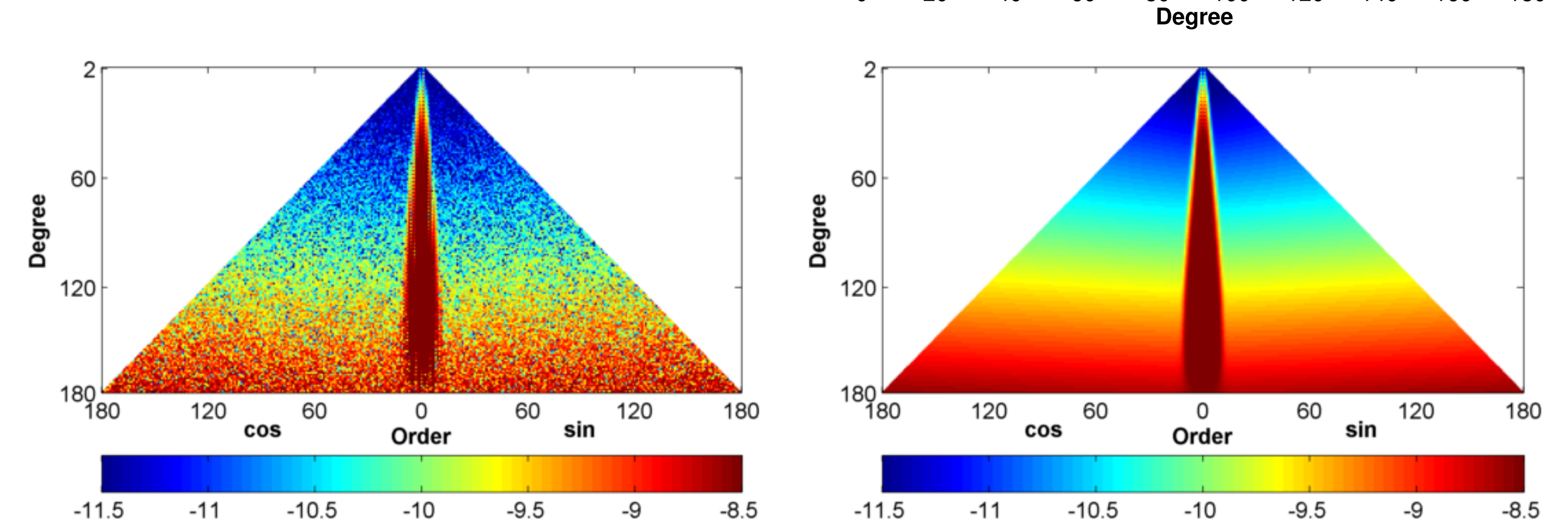

Fig. 6: Gravity field solution from clock data with a noise level of 1.0×10^{-18}. Coefficient differences w.r.t. EIGEN-6c4 (left) and the formal errors (right), in logarithm scale.

Potential for deriving the temporal gravity field

A simulated orbit was used

- Altitude: 350 km;
- Inclination: 89.5°;
- Repeat cycle: 377 revolutions in 24 nodal days.

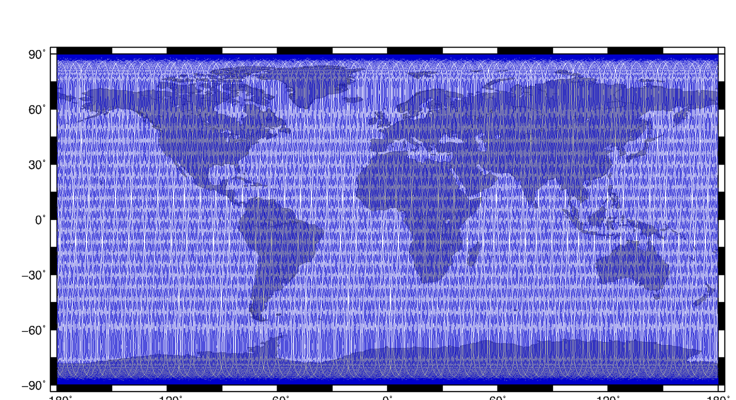

Fig. 7: Ground tracks of the simulated orbit.

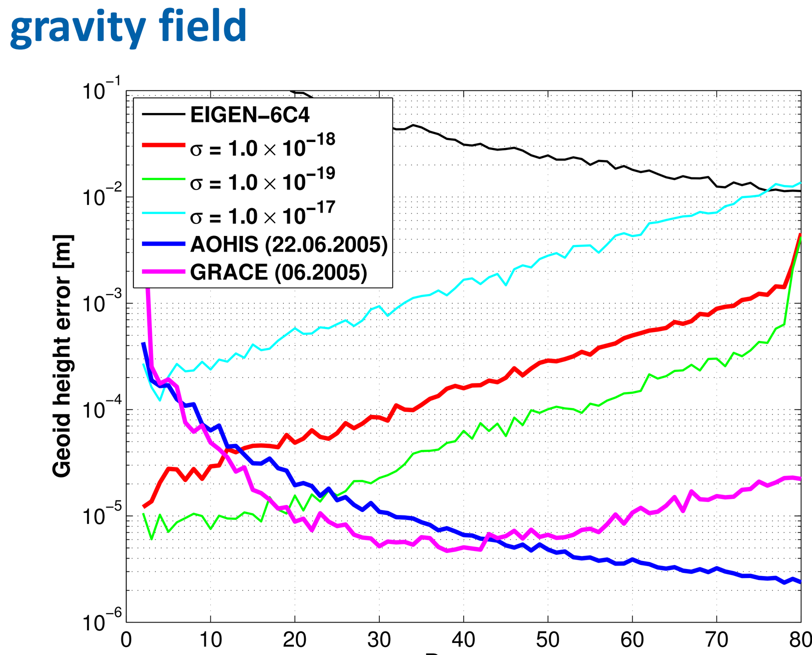

Fig. 8: Comparison of gravity field errors and the temporal gravity signal. The AOHIS (Atmosphere, Ocean, Hydrology, Ice and Solid earth tide) model is used for the forward modelling of the temporal gravity signal. The GRACE monthly solution (06.2005) is taken as a reference.

Cold Atom Interferometry (CAI) gradiometer

Compared to the electrostatic one, the CAI gradiometer has

- better sensitivity: 1.0 – 5.0 $\mathrm{mE}/\sqrt{\mathrm{Hz}}$;
- wide spectral range: flat noise down to very low frequency.

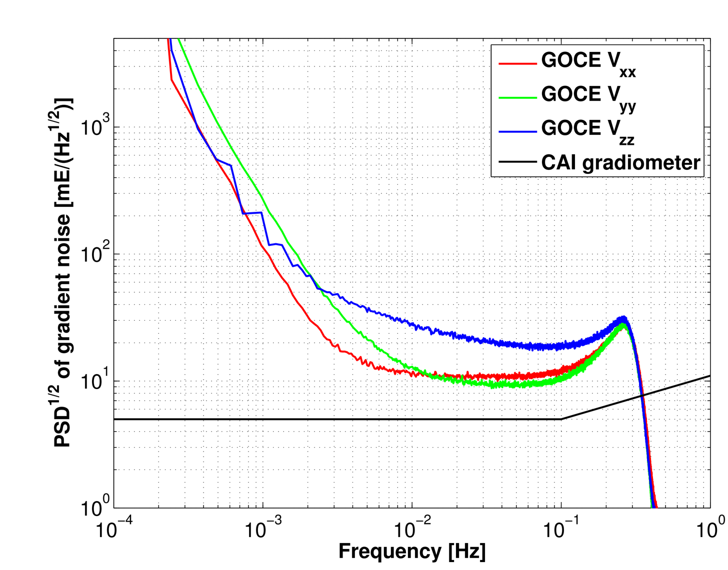

Fig. 9: Spectral noise behavior of the CAI gradiometer, compared to the GOCE gravity gradients.

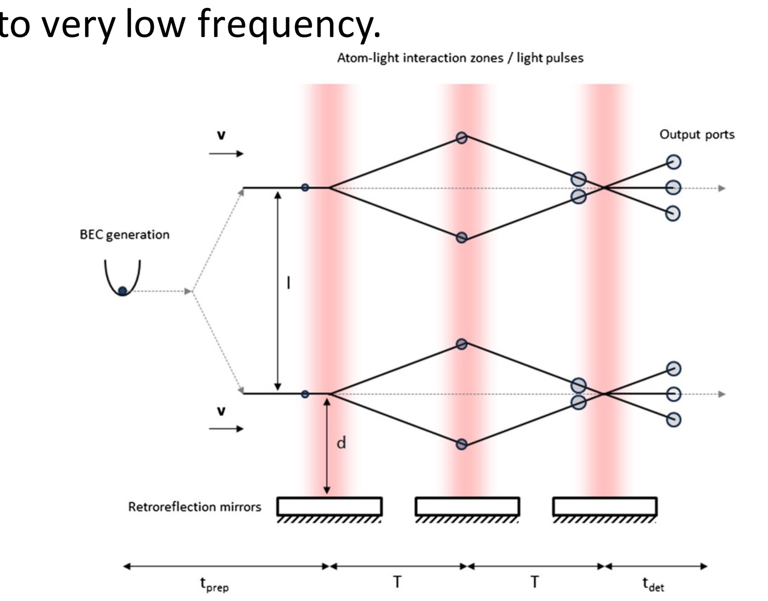

Fig. 10: Atom interferometry scheme for gradiometric measurements as proposed in Carraz et al. (2014).

Input for simulation

- Orbit: GOCE, 71 days (1st Mar. – 10th May, 2013), 2s;
- Model: EIGEN-6c4, d/o 360;
- Noise: white, 5.0 $\mathrm{mE}/\sqrt{\mathrm{Hz}}$.

Two pointing modes

- Nadir:
  - one axis: V_{yy};
  - three axes (tilting mirror): V_{xx}, V_{yy}, V_{zz};
- Inertial: V_{xx}, V_{yy}, V_{zz}.

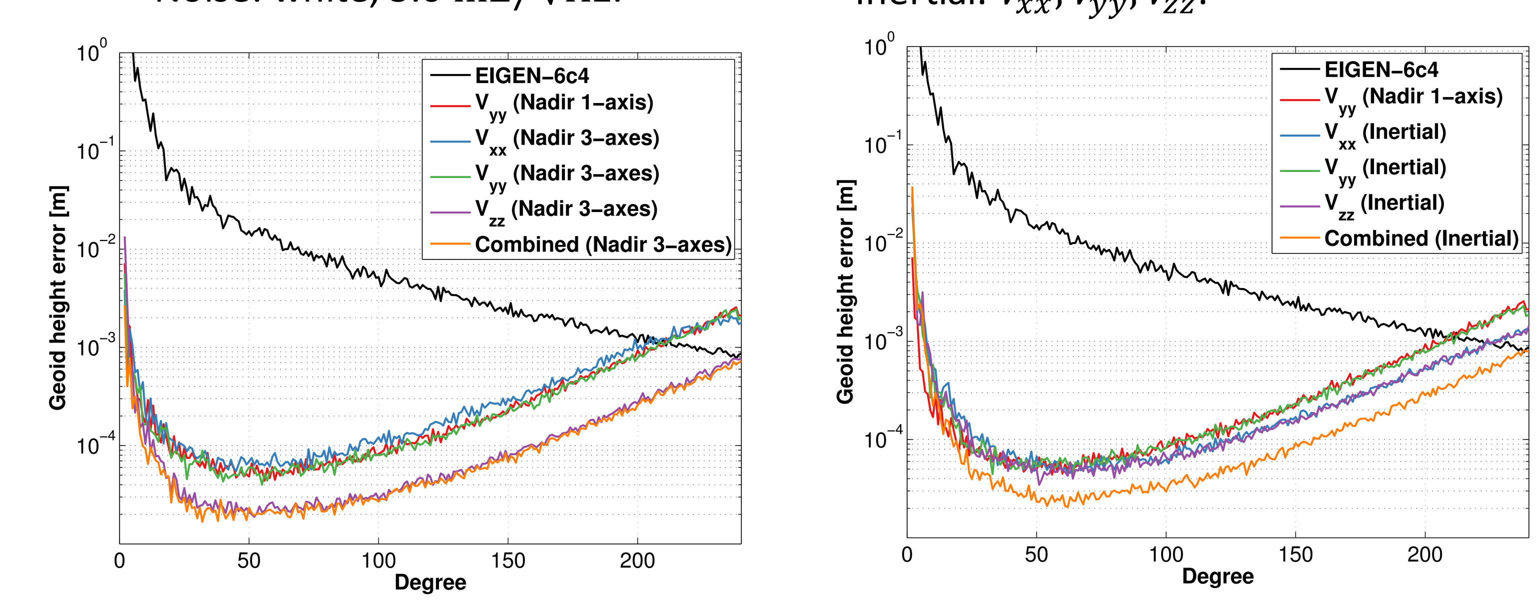

Fig. 11: Degree medians of gravity field coefficient differences w.r.t. EIGEN-6c4, in terms of geoid height. The left figure shows results in the nadir mode while the right figure shows results in the inertial mode. All CAI models were recovered up to d/o 240.

Combined analysis

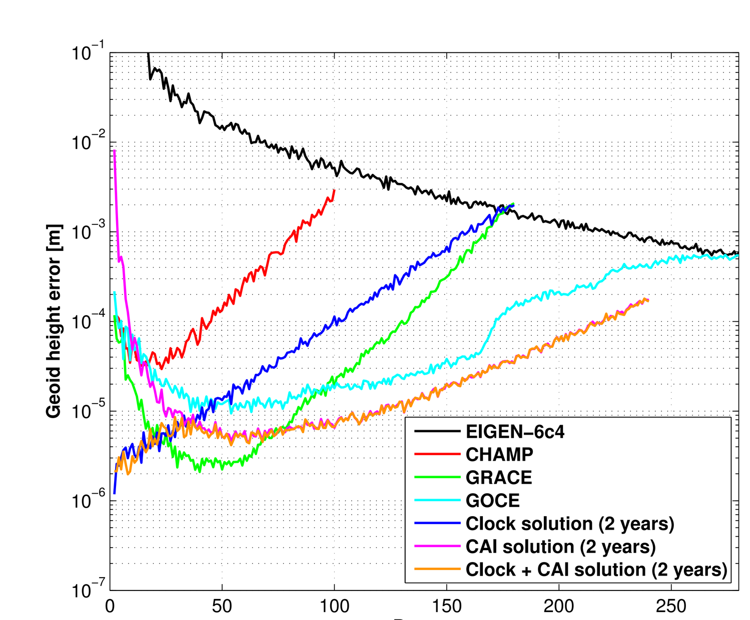

Fig. 12: Degree medians of gravity field coefficient differences w.r.t. EIGEN-6c4, in terms of geoid height. To compare with the official CHAMP, GRACE and GOCE gravity field solutions, we scaled the clock, CAI and their combined solutions to two years.

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Conclusions

- Clocks deliver scalar observations (not affected by attitude errors), and improve the long-wavelength gravity field, e.g., below d/o 30;
- Clocks show a good potential to detect temporal gravity field signals at very low degrees;
- CAI gradiometry in 3-axes modes outperforms GOCE by more than a factor of 5.

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