Multiplicative noise and intermittency in bedload sediment transport

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Abstract

Sediment transport in rivers near the threshold of grain motion is characterized by rare but large transport events. This intermittency makes it difficult to relate average sediment flux to average flow conditions, or to define an unambiguous threshold for grain entrainment. Although intermittent sediment transport can be observed and characterized, its origins are unclear. In this study we investigate bedload sediment transport near the threshold of grain motion in an experimental flume to examine the origins of intermittency. We apply image-processing techniques to high-speed video of grains in a narrow flume, which allows us to track individual particles and measure statistics of particle motion. Bedload sediment transport near the threshold of grain motion is very low, allowing us to approximate the time evolution of the sediment flux via a polynomial expansion, including a linear growth rate and a nonlinear term which saturates the growth. We introduce a noisy coefficient to the linear growth rate term ("multiplicative noise"), rather than adding the noise to the equation, to model the inherent fluctuations in the system. We demonstrate that multiplicative noise near the threshold of grain motion can account for the observed intermittency. We use analytical results from bifurcation theory in the presence of multiplicative noise to analyze our experimental results, quantifying the noise responsible for the intermittency and calculating the critical shear stress for grain entrainment in a novel way that is consistent with the physics of grain motion at low transport stages.

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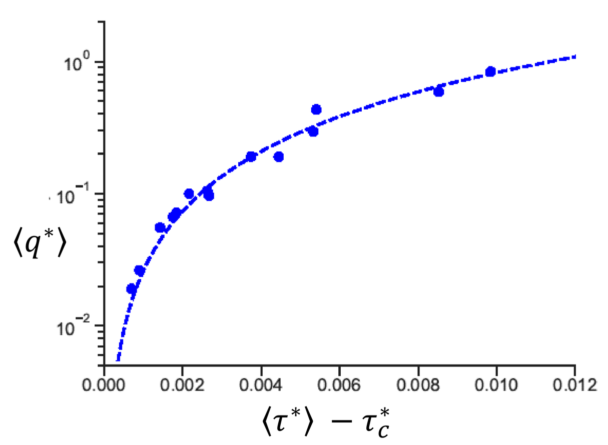
Key Points

- Sediment transport near the threshold of motion is intermittent; it comes in short, intense bursts. This makes it difficult to measure the average sediment flux and define the threshold of motion itself.
- We use bifurcation theory and the concept of multiplicative noise to understand and describe the intermittency.
- Applying this to a set of flume experiments[1], we find a new way of measuring the critical shear stress, τ_c^*, and a way of predicting when intermittency will dominate sediment transport.

Bedload Transport

- Want to predict mean (dimensionless) volume flux, $\langle q^* \rangle$, given a mean shear stress, $\langle \tau^* \rangle$ at the bed.

Himachal Pradesh, India



Experimental results [1]

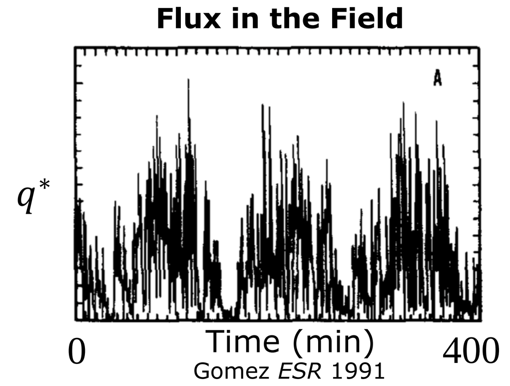
- For example, most common and successful version is the 3/2 law:

$$\langle q^* \rangle \propto (\langle \tau^* \rangle - \tau_c^*)\left(\sqrt{\langle \tau^* \rangle} - \sqrt{\tau_c^*}\right)$$

Intermittency

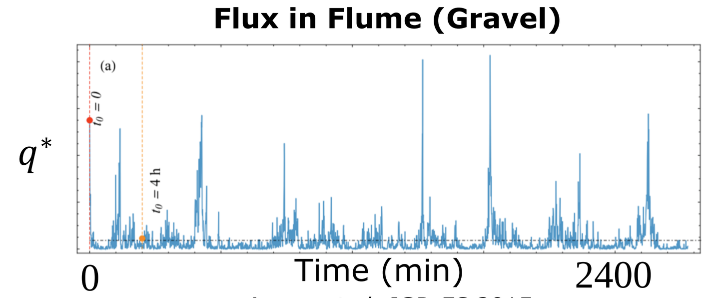
- Experimental and field measurements show the presence of intermittency at low transport stages.
- Implications include:
  - Can't measure $\langle q^* \rangle$ accurately over short intervals.
  - Hard to define τ_c^*
- Models of average bedload flux don't account for intermittency.
  - Intermittency can cause *breakdown* of average laws.

Flux in the Field



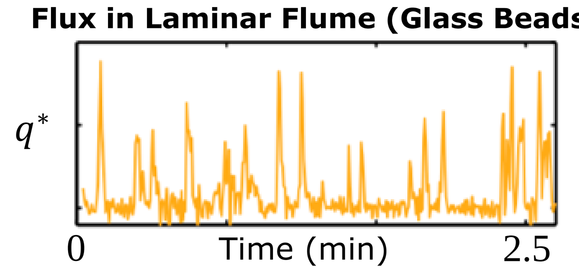
Gomez *ESR* 1991

Flux in Flume (Gravel)



Ancey *et al.* *JGR:ES* 2015

Flux in Laminar Flume (Glass Beads)



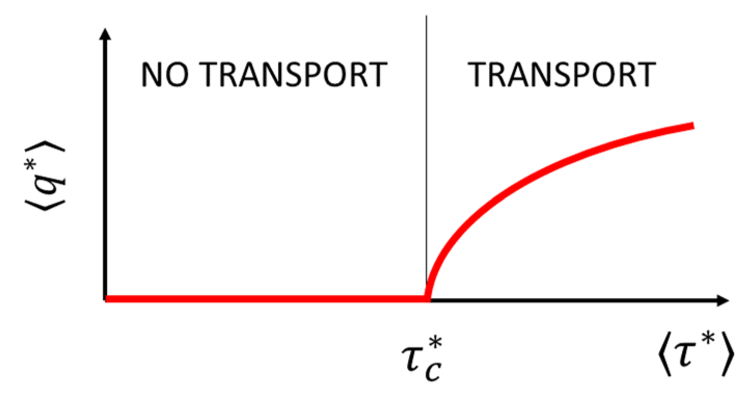
Houssais *et al.* *Nat. Comm.* 2016

Bifurcations and Multiplicative Noise

Bedload transport as a bifurcation:
- For $\tau^* < \tau_c^*$, no transport: $\langle q^* \rangle = 0$
- For $\tau^* \geq \tau_c^*$, transport: $\langle q^* \rangle > 0$

Close to the threshold, $q^* \ll 1$, and:

$$\frac{dq^*}{dt} = g(q^*, \langle \tau^* \rangle, \tau_c^*, \ldots) \approx (\langle \tau^* \rangle - \tau_c^*)q^* - NL(q^*)$$

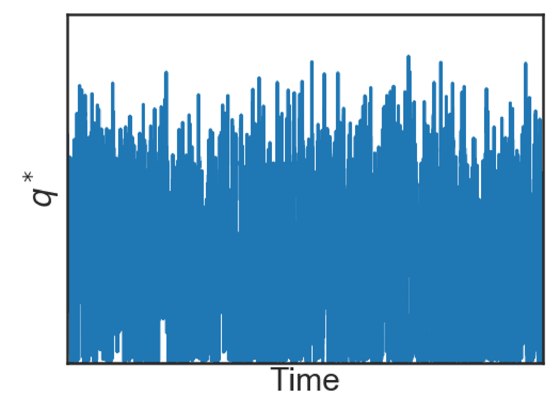


Bifurcation diagram schematic for bedload transport

However, sediment transport is noisy (turbulence, collisions...). How do we include the noise?

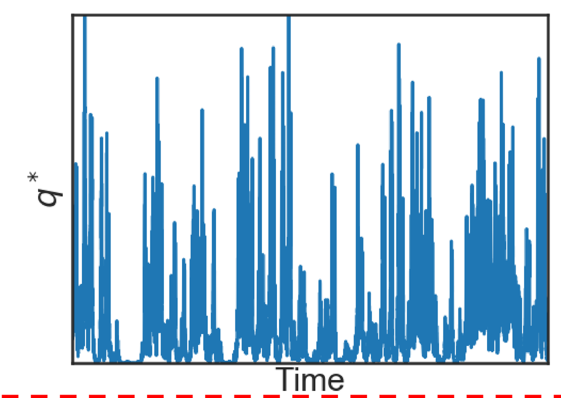
<u>Additive Noise</u>

$$\frac{dq^*}{dt} \approx (\langle \tau^* \rangle - \tau_c^*)q^* - NL(q^*) + Noise$$



<u>Multiplicative Noise</u>

$$\frac{dq^*}{dt} \approx (\langle \tau^* \rangle - \tau_c^* + Noise)q^* - NL(q^*)$$



Multiplicative noise causes what's called on-off intermittency [2] when $\langle \tau^* \rangle \to \tau_c^*$. Statistical predictions [3]:

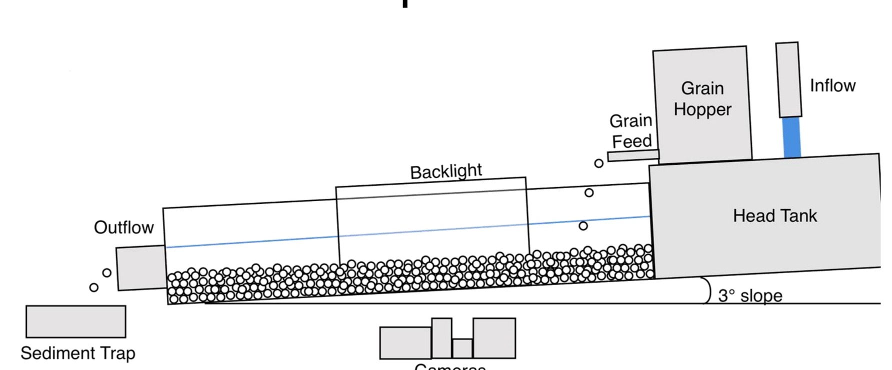
a) Intermittent if $\langle \tau^* \rangle - \tau_c^* < S$, and S = autocorrelation of the noise.

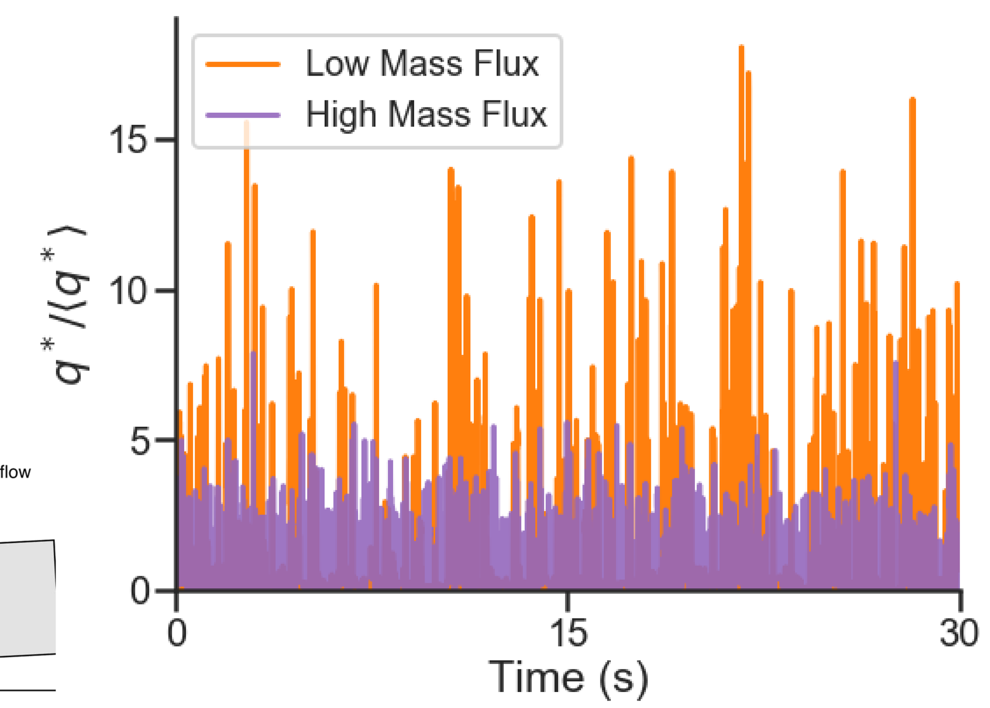
b) Waiting time between large events is power law with exponent -3/2

c) Distribution of possible mass flux also power law, with exponent -1

Laboratory Flume Experiments

- Performed at SFU in the Venditti lab.
- 5mm glass beads, ~10mm-wide flume.
- Constant water flux, varied mass flux input.

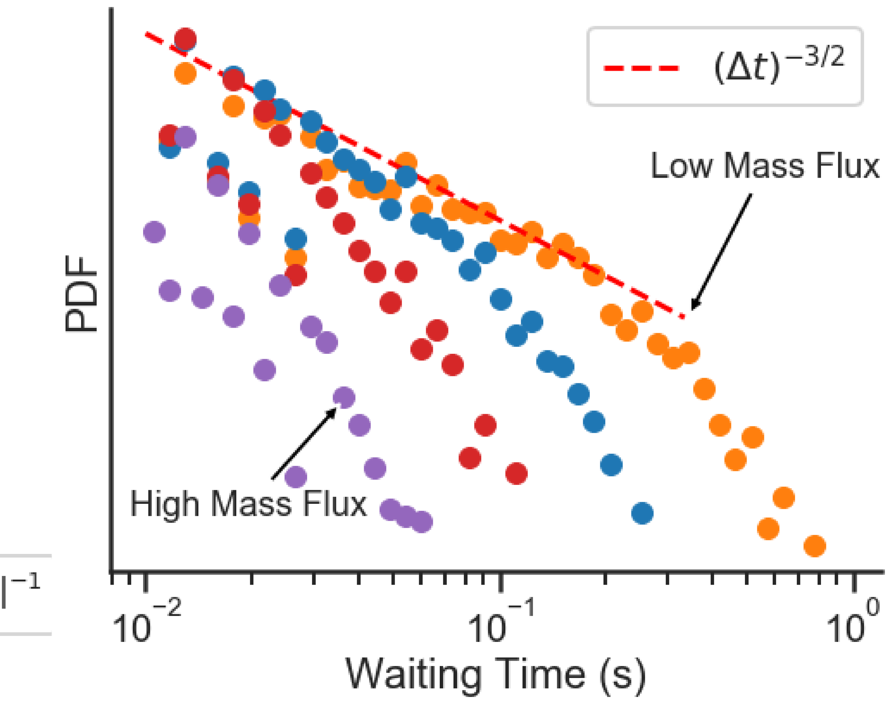


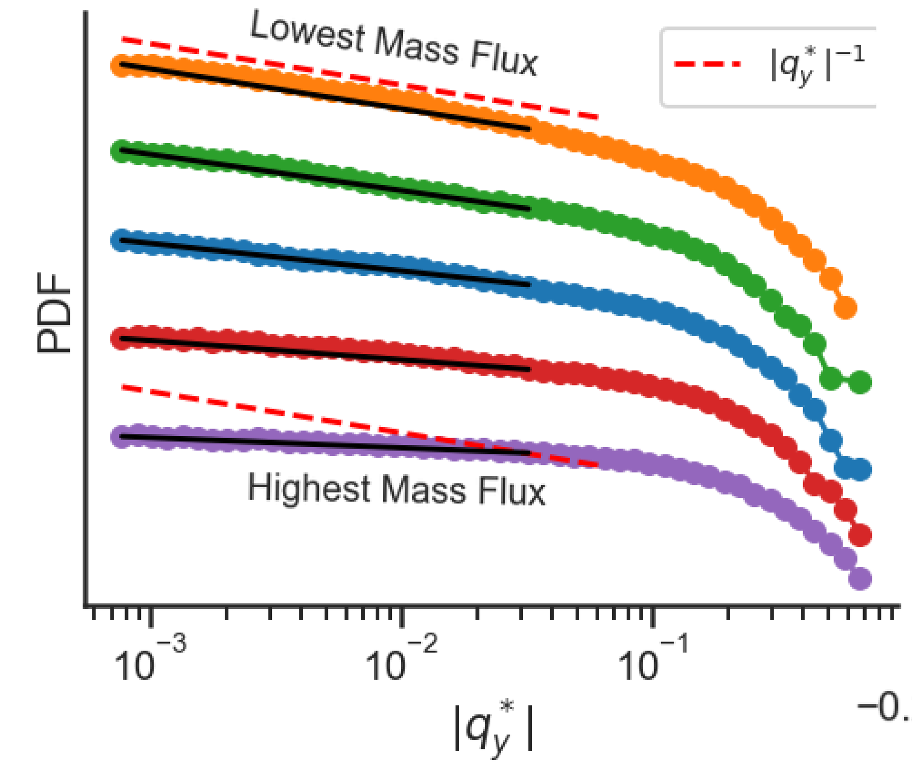


Experimental Time Series [1]

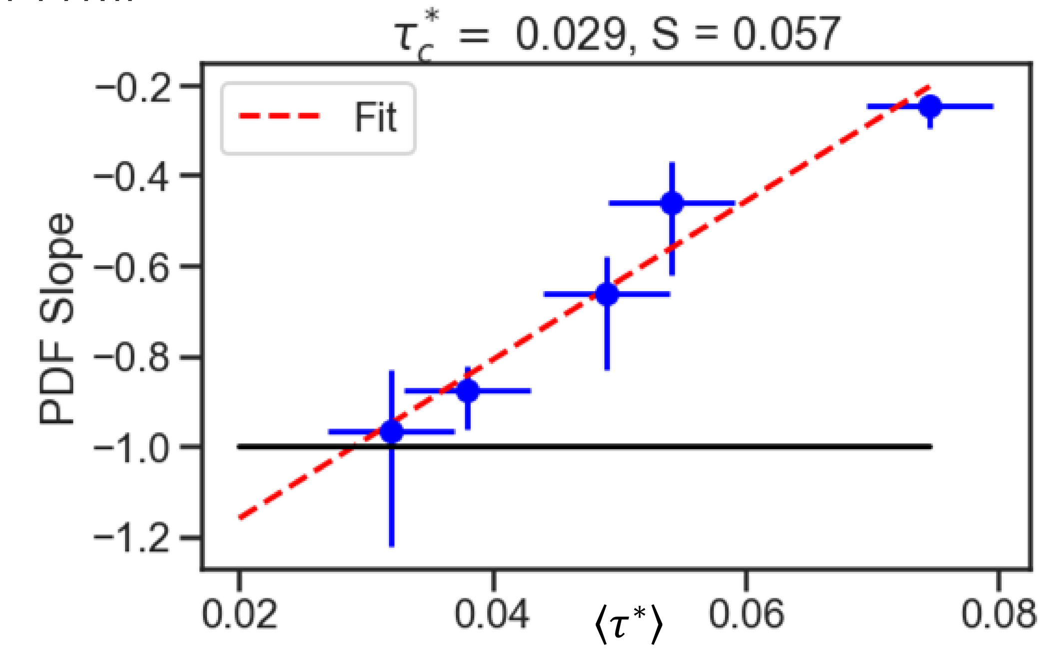
Testing Predictions

- Prediction:
$$\mathrm{PDF}_{q_0^*}(\Delta t) \propto (\Delta t)^{-\frac{3}{2}}$$
- Cutoff depends on threshold proximity





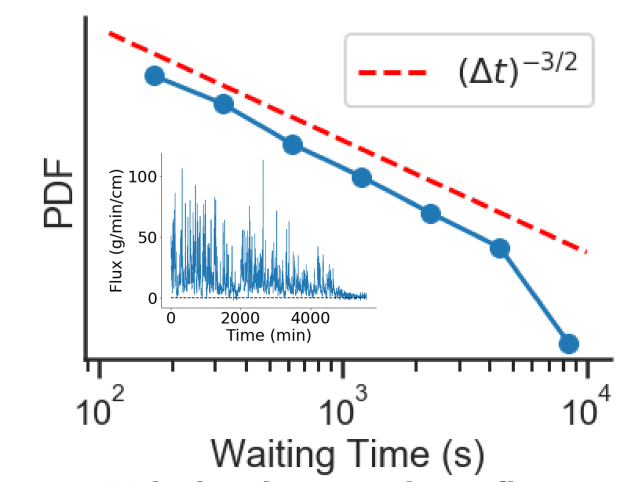
- Prediction:
$$\mathrm{PDF}(q^*) \propto (q^*)^{\frac{\langle \tau^* \rangle - \tau_c^*}{S} - 1}$$
- Slope measures threshold proximity



- Calculated τ_c^* in a novel way.
- Extracted property of noise, S.

Implications and Future Work

- Want to confirm with other systems
  - Flume with natural grains (in progress)
  - Field data (preliminary, see figure)
- Future questions/studies:
  - How does S vary between systems?
  - Motivate simple ODE with physics?



Wait time between large flux, Nelson *et al.* *JGR* (114) 2009

References

[1] Eric Deal *et al.* EP41B-2650: Observing the role of grain shape on bedload transport in paired flume experiments and numerical simulations. AGU Fall Meeting, 2018.
[2] Heagy, J. F., Platt, N., & Hammel, S. M. *Phys. Rev. E*, 49(2):1140-1150, 1994.
[3] Aumaître, S., Pétrélis, F., & Mallick, K. *Phys. Rev. Lett.* 95:064101, 2005