

# Laminar and Turbulent Lava Flow in Sheets, Channels, and Tubes: Estimating Terrestrial and Planetary Lava Flow Rates

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## Abstract

To date, actively flowing lava has only been observed on Earth and on Jupiter’s moon Io. This lack of observation means that for the vast majority of volcanic systems in the Solar System, solidified lava-flow morphologies are used to infer important information about eruption and emplacement parameters. These include: lava supply rate, lava composition, lava rheology, and determination of laminar or turbulent emplacement regimes. Commonly used models that relate simple lava flow morphologic properties (e.g., width, thickness, length) to emplacement characteristics are based on assumptions that are readily misinterpreted. For example, the simplifying assumption of fully turbulent lava flow allows for a thermally mixed flow interior, but ignores the lava properties that naturally work to suppress full turbulence (such as thermal boundary layers encasing active lava flows, and a temperature-dependent lava rheology). However, full turbulence in silicate lava flows erupted into environments that have temperatures lower than the lava solidification temperature requires a rare combination of characteristics. We model Bingham Plastic, Newtonian, and Herschel-Bulkley fluids in rectangular channels, tubes, and sheets with computational fluid dynamics (COMSOL) software to obtain flow solutions and general flow rate equations and compare them to field measurements of volcanic velocity and flow rates. We present these as more realistic alternatives to older simpler rate-from-morphology models. We find that several lava rheology properties work together to delay the onset of turbulence as compared to isothermal Newtonian materials, and that while turbulent lavas flows certainly exist, they are not as prevalent as the published literature might indicate. Results obtained from models that assume full turbulence in silicate flows on the terrestrial planets should therefore be interpreted cautiously.

# Laminar and Turbulent Lava Flow in Sheets, Channels, and Tubes: Estimating Terrestrial and Planetary Lava Flow Rates

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**Terrestrial Approach:**  
Compute or predict lava flow length and width from basic flow properties.

**Planetary Approach:**  
Compute or predict basic flow properties from lava flow length and width.

## Abstract

To date, actively flowing lava has only been observed on Earth and on Jupiter's moon Io. This lack of observation means that for the vast majority of volcanic systems in the Solar System, solidified lava-flow morphologies are used to infer important information about eruption and emplacement parameters. These include: lava supply rate, lava composition, lava rheology, and determination of laminar or turbulent emplacement regimes. Commonly used models that relate simple lava flow morphologic properties (e.g., width, thickness, length) to emplacement characteristics are based on assumptions that are readily misinterpreted. For example, the simplifying assumption of fully turbulent lava flow allows for a thermally mixed flow interior, but ignores the lava properties that naturally work to suppress full turbulence (such as thermal boundary layers encasing active lava flows, and a temperature-dependent lava rheology). However, full turbulence in silicate lava flows erupted into environments that have temperatures lower than the lava solidification temperature requires a rare combination of characteristics. We model Bingham Plastic, Newtonian, and Herschel-Bulkley fluids in rectangular channels, tubes, and sheets with computational fluid dynamics (COMSOL) software to obtain flow solutions and general flow rate equations and compare them to field measurements of volcanic velocity and flow rates. We present these as more realistic alternatives to older simpler rate-from-morphology models. We find that several lava rheology properties work together to delay the onset of turbulence as compared to isothermal Newtonian materials, and that while turbulent lavas flows certainly exist, they are not as prevalent as the published literature might indicate. Results obtained from models that assume full turbulence in silicate flows on the terrestrial planets should therefore be interpreted cautiously.

## Analytic Flow Rate Models

Hulme model: (Isothermal laminar Bingham Approximation)

$$Y_B = 2\rho g W_L \sin^2 \theta$$

$Y_B$  = Bingham yield stress  
 $\rho$  = flow density  
 $g$  = acceleration of gravity  
 $W_L$  = Levee width  
 $\theta$  = slope

→ This approximation (e.g. Hulme, 1976) relies on problematic assumptions and should be replaced with an exact or empirical solution (e.g. Skelland 1967, Deane and Sakimoto, 1997; Burger et al. 2015; and others).

Jeffries Equation: (Isothermal laminar Newtonian Approximation)

$$u = \frac{h^2 g \rho \sin \theta}{B \eta}$$

$u$  = flow velocity  
 $h$  = flow depth  
 $g$  = acceleration of gravity  
 $\rho$  = density  
 $B$  = geometry parameter  
 $\eta$  = fluid viscosity

→ This approximation (Jeffries, 1925) should be replaced with the appropriate exact solution. See, for example, White (2006).

Isothermal laminar Bingham Flow Examples

→ Numerous exact and empirical solutions:  
Skelland, 1967, circular tube, parallel plates  
Burger et al (2015), and many others: rectangular channel  
Deane and Sakimoto (1997) Parabolic Channel  
...etc...

## Exporting Models to Planetary Flows

- Planetary volcanology approach inverts the terrestrial approach:
  - Planetary ... often predicting flow properties from flow dimensions and shape rather than flow dimensions from flow properties.
- Planetary flows do not have geochemistry or geothermometry constraints that may be available for terrestrial flows. Ambient conditions must be considered.
- Computational approaches modeling flows with changed ambient conditions for planetary flows are expected to yield improved results compared to exporting vintage terrestrial approximations. Computational approaches can yield empirical equations appropriate for specific model/planet conditions.

## References

Blair, D. M., J. Chappaz, R. Saad, C. Milroy, A. Bobet, H.L. Mahesh, K. C. Howell, A. M. Freed, (2017). The structural stability of linear lava tubes, *Earth and Planetary Science Letters*, 469, pp. 306-316.

Blair, D.M., R.C. Armstrong, and B. Hargrove, (2017) Dynamics of Polymeric Liquids, 6th ed. John Wiley, NY.

Blair, R.R., G.C. Dai, and B.A. Yarnas, (1985) The rheology and flow of viscoplastic materials, *Reviews in Chemical Engineering*, 1(1), 1-70.

Blair, R.R., E. Stevens, and J.N. Ligonelli, (1991) Transport Phenomena, 2nd ed. John Wiley and Sons, NY.

Bishop, R.C., S.M. Baloga, and J. Forster, (1986) Modeling granular flows on an inclined plane, *Journal of Geophysical Research*, 91, (B5), 11565-11577.

Blair, R.R., E. Stevens, and J.N. Ligonelli, (1991) Transport Phenomena, 2nd ed. John Wiley and Sons, NY.

Car, M.L., R. Gentry, K.B. Blanton, J.E. Goetz, and J.B. Murray, (1977) Some Mars-like volcanic features as viewed from the Viking Orbiters, *Journal of Geophysical Research*, 82, 3985-3915.

Cannata, G., (1997) *Supervolcanic eruptions and their hazards*, Wiley, New York.

COMSOL Multiphysics v. 5.4a (2018 release) www.comsol.com/COMSOL\_AB\_Software, Sweden.

Deane, S.E., S. Sakimoto, and B. Hargrove, (1997) Structural evolution of a lava flow and its implications for the flow rate, *Journal of Geophysical Research*, 102, 18,009-18,018.

Deane, S.E., S. Sakimoto, and B. Hargrove, (1998) Knowledge flow models of a Bingham liquid: implications for lava flows, *Journal of Volcanology and Geothermal Research*, 30, 385-325.

Duggan, M., A. Pamberton, and K. F. Johnson, (1995) A model for the formation of lava tubes by solidification of a lava flow, *Journal of Geophysical Research*, 100, 9455-9458.

Duggan, M., P. O'Hara, A. Tuffen, (2002) Temperature distribution and annual lava tube, *Journal of Geophysical Research*, 107, 43-51, 10.1029/2001JB003084.

Gentry, G.R., (1971) Observations of actively flowing lavas and associated structures, *Journal of Geophysical Research*, 76, 2141-2144.

Goetz, J.B., (2017) *Planets and Processes: Subsurface Lava Flow Morphologies*, *Journal of Volcanology and Geothermal Research*, 342, 3-12, 10.1016/j.jvolgeot.2017.04.022.

Goetz, J.B., (1999) *Understanding Bingham fluid flow*, *Journal of Geophysical Research*, 104, 21,229-21,234.

Harris, M., J.L. (2013), *Lava Flows: Chapter 5 in Modeling Volcanic Processes: The Physics and Mathematics of Volcanism*, Cambridge University Press, 978-0-521-89543-9.

Harris, M., (1974) The emplacement of lava flows, *Geology*, 2, 203-205.

Harris, M., (1976) The determination of the rheological properties and effusion rate on an Olympus Mons Lava, *Earth and Planetary Science Letters*, 27, 207-213.

Harris, M., and G. S. Ellenor, (1976) The emplacement of the Pleistocene Lava Flows of the Mount St. Helens, *Journal of Geophysical Research*, 81, 225-234, 1977.

Kilb, H. (1925) The flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1926) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1927) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1928) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1929) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1930) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1931) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1932) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1933) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1934) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1935) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1936) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1937) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1938) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1939) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1940) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1941) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1942) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1943) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1944) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1945) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1946) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1947) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1948) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1949) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1950) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1951) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1952) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1953) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1954) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1955) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1956) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1957) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1958) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1959) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1960) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1961) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1962) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1963) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1964) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1965) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1966) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1967) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1968) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1969) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1970) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1971) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1972) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1973) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1974) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1975) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1976) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1977) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1978) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1979) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1980) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1981) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1982) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1983) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1984) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1985) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1986) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1987) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1988) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1989) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1990) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1991) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1992) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1993) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1994) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1995) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1996) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1997) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1998) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (1999) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (2000) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (2001) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (2002) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (2003) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (2004) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (2005) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (2006) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (2007) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (2008) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (2009) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (2010) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (2011) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (2012) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (2013) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (2014) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (2015) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (2016) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (2017) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

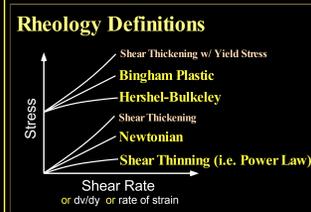
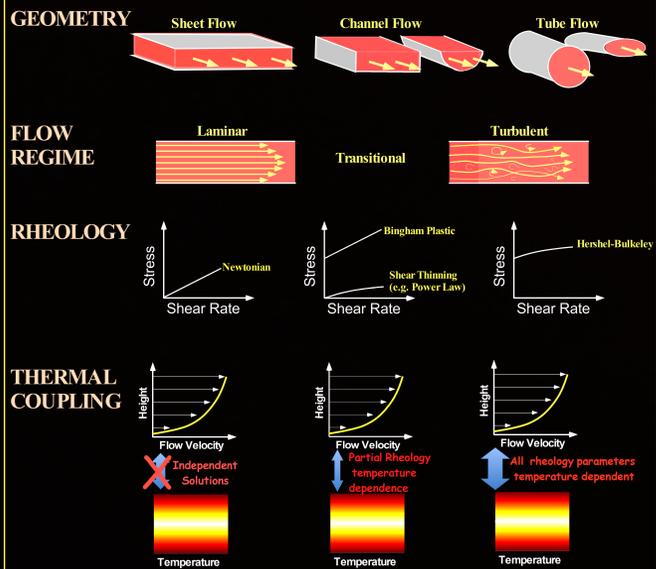
Kilb, H., (2018) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (2019) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

Kilb, H., (2020) A preliminary note on the flow of water in an inclined channel of rectangular section, *Phil. Magazine*, 49, 793-807.

## Model Approaches

LESS COMPLEX → MORE COMPLEX

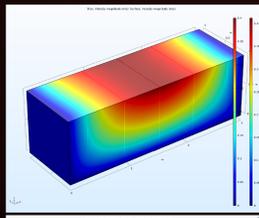


**Common Approach:**  
For: Observed Flow Dimensions Model: Coupled Rheology and Flow Rate  
On Earth, rheology is often further constrained with geochemistry or geothermometry

## Computational Flow Rate Model Examples

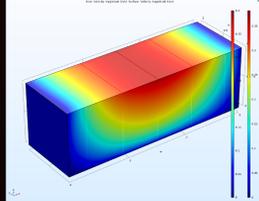
All computational models are done with COMSOL Multiphysics 5.4 (2018) using the Computational fluid dynamics (CFD) module, the Heat Transfer Module, and (where applicable) the Material Library Module. See www.comsol.com for a more complete description of the computational capabilities of each module and the base multiphysics package.

Isothermal laminar Newtonian flow in a rectangular channel:



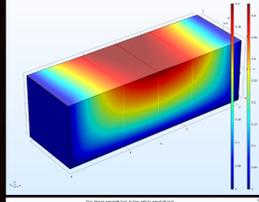
- parabolic velocity profile
- expected turbulence transition

Isothermal power law thinning flow in a rectangular channel:



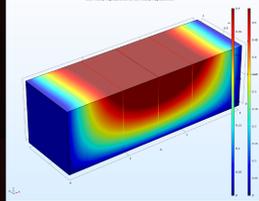
- more flow in the boundary layers
- slower center velocity
- delayed turbulence transition

Isothermal power law thickening flow in a rectangular channel:



- less boundary layer flow
- faster center velocity
- delayed turbulence transition

Isothermal Bingham thinning flow in a rectangular channel:



- substantial fast center plug
- thin boundary layer
- delayed turbulence transition

## Discussion and Conclusions

- The Hulme and Jeffries approximation approaches for estimating planetary flow properties should be retired, since they are demonstrably unreliable, and we have vastly improved analytic AND computational approaches that yield more tightly constrained and consistent results.
- We do not yet adequately understand the effects of temperature-dependent rheology, composition, and ambient conditions on terrestrial or planetary flows. So:
  - Inferring composition variations from flow morphology is fraught with pitfalls
  - Inferring laminar or turbulent emplacement from flow morphology and multiple simplifying assumptions has significant potential for incorrect results.
- With current computational tools, we can construct semi-empirical relationships as well as self-contained model applications that are specific to flow conditions and planetary conditions
  - ...increasing our understanding of planetary flow properties AND general lava flow processes under different ambient conditions..